Preprocessing QBF: Failed Literals and Quantified Blocked Clause Elimination

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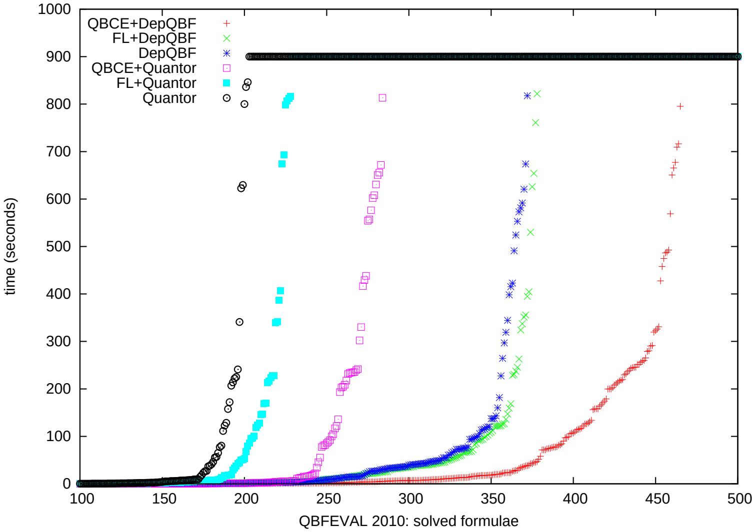
Preprocessing Techniques for Quantified Boolean Formulae (QBF)

- Failed literals (FL) and quantified blocked clause elimination (QBCE).
- Positive effects on search- and elimination-based solvers.



Overview

Part 1: Preliminaries

- From propositional logic (SAT) to QBF.
- QBF semantics.

Part 2: Failed Literal Detection (FL)

- Paper submitted to SAT'11.
- Necessary assignments and QBF models.

Part 3: Quantified Blocked Clause Elimination (QBCE)

- Paper submitted to CADE'11.
- From BCE for SAT to QBCE for QBF.

Part 1: Preliminaries

Propositional Logic (SAT):

- Our focus: formulae in conjunctive normal form (CNF).
- Set of Boolean variables $V := \{x_1, \ldots, x_m\}$.
- Literals $l := v$ or $l := \neg v$ for $v \in V$.
- Clauses $C_i := (l_1 \vee \ldots \vee l_{k_i})$.
- CNF $\phi := \bigwedge C_i$.

Quantified Boolean Formulae (QBF):

- Prenex CNF: quantifier-free CNF over quantified Boolean variables.
- PCNF $Q_1 S_1 \ldots Q_n S_n.\ \phi$, where $Q_i \in \{\exists, \forall\}$, scopes S_i.
- Scope S_i: set of quantified variables.
- $Q_i S_i \leq Q_{i+1} S_{i+1}$: scopes are linearly ordered.

Example

Clauses (CNFs) are sets of literals (clauses).
A CNF: $\{x, \overline{y}\}, \{\overline{x}, y\}$ and a PCNF: $\forall x \exists y.\ \{x, \overline{y}\}, \{\overline{x}, y\}$.

SAT Semantics

Assignment Trees (AT):

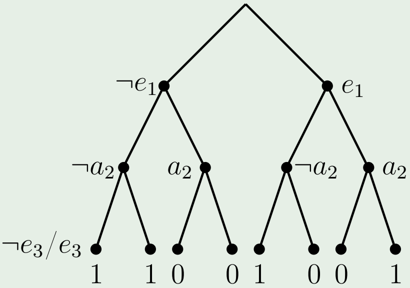
- Assignment $A : V \rightarrow \{\mathit{true}, \mathit{false}\}$ maps variables to truth values.
- Paths from root to a leaf in AT represent assignments.
- Nodes along path (except root) assign truth values to variables.

CNF-Model:

- A path in the assignment tree of a CNF ϕ which satisfies all clauses.
- CNF ϕ is satisfiable iff it has a CNF-model m: $m \models \phi$.

Example

$$\begin{aligned}\phi \quad := \quad & \{e_1, \neg a_2, e_3\}, \\ & \{e_1, \neg a_2, \neg e_3\}, \\ & \{\neg e_1, a_2, \neg e_3\}, \\ & \{\neg e_1, \neg a_2, e_3\}\end{aligned}$$



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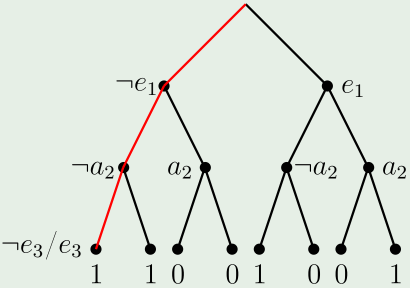

Semantics: From SAT to QBF

PCNF-Model: $\psi := Q_1 S_1 \ldots Q_n S_n.\ \phi$

- An (incomplete) AT where *every* path is a CNF-model of CNF part ϕ.
- Restriction: nodes which assign $\forall$-variables have exactly one sibling.
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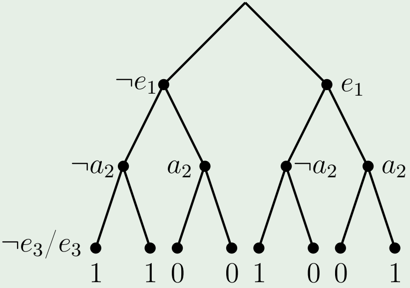

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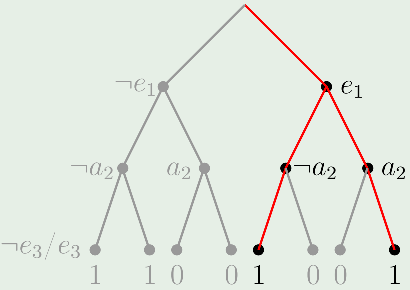

QBF Inference Rules (1/5)

Definition (Assignments of literals)

Given a PCNF ψ, the *assignment of a literal l* yields the formula $\psi[l]$ where clauses $Occs(l)$ and literals $\neg l$ in $Occs(\neg l)$ are deleted.

Example

$$\begin{aligned}
\psi &:= \exists e_1 \forall a_2 \exists e_3, e_4.\ \phi \\
\phi &:= \{e_1, a_2, e_3, e_4\}, \\
&\quad \{e_1, a_2, \neg e_4\}, \\
&\quad \{\neg e_1, e_3, \neg e_4\}, \\
&\quad \{\neg a_2, \neg e_3\}
\end{aligned}$$

$\psi[e_4]$

Definition (Universal Reduction)

Given a clause C, $UR(C) := C \setminus \{l_u \in L_\forall(C) \mid \nexists l_e \in L_\exists(C), l_u < l_e\}$.

Example

$$\psi := \exists e_1 \forall a_2 \exists e_3, e_4.\ \phi$$

$$\phi := \{e_1, a_2\}, \{\neg e_1, e_3\}, \{\neg a_2, \neg e_3\}$$

$UR(\{e_1, a_2\})$

Definition (Pure Literal Rule)

Given a PCNF ψ, a literal l where $Occs(l) \neq \emptyset$ and $Occs(\neg l) = \emptyset$ is *pure*: if $q(l) = \exists$ then $\psi \equiv \psi[l]$, and if $q(l) = \forall$ then $\psi \equiv \psi[\neg l]$.

Example

$$\psi := \exists e_1 \forall a_2 \exists e_3, e_4.\ \phi$$

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Variable a_2 is pure: $\psi[a_2]$ (shortening clauses).

QBF Inference Rules (4/5)

Definition (Unit Clause Rule)

Given a PCNF ψ. A clause $C \in \psi$ where $UR(C) = \{l\}$ is *unit* and $\psi \equiv \psi[l]$.

Example

$$\psi := \exists e_1 \forall a_2 \exists e_3, e_4.\ \phi$$

$$\phi := \{e_1\}, \{\neg e_1, e_3\}, \{\neg e_3\}$$

Clauses $\{e_1\}$ and $\{\neg e_3\}$ are unit: $\psi[e_1][\neg e_3]$.

QBF Inference Rules (5/5)

Definition (Boolean Constraint Propagation)

Given a PCNF ψ and a literal x called *assumption*. Formula $BCP(\psi, x)$ is obtained from $\psi[x]$ by applying UR, unit clause and pure literal rule.

Example

$$\psi := \exists e_1 \forall a_2 \exists e_3, e_4.\ \phi$$

$$\phi := \{\}$$

Empty clause derived from assumption e_4:

$\emptyset \in BCP(\psi, e_4)$.

Part 2: Failed Literal Detection (FL)

Definition

Given PCNF ψ and $x_i \in V$. Assignment $x_i \mapsto t$, where $t \in \{\mathit{false}, \mathit{true}\}$, is necessary for satisfiability of ψ iff $x_i \mapsto t$ is part of *every* path in every PCNF-model of ψ.

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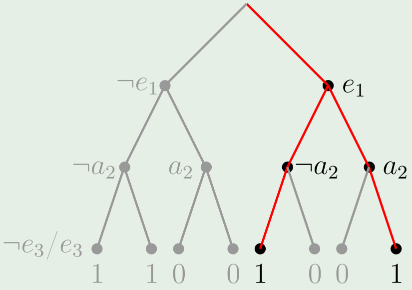
- $e_1 \mapsto$ *true* is necessary for satisfiability of ψ.



GOAL: Detection of (Subset of) Necessary Assignments in QBFs.

- Exponential reduction of search space.

Models and Necessary Assignments

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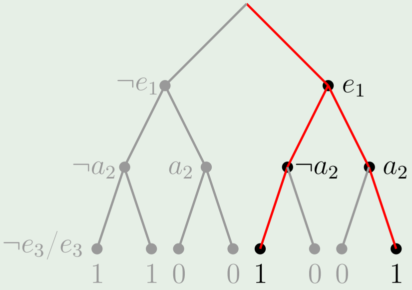

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Failed Literal Detection (FL) for SAT:

- BCP-based approach to detect subset of necessary assignments.
- Def. failed literal x for CNF ϕ: if $\emptyset \in BCP(\phi, x)$ then $\phi \equiv \phi \wedge \{\neg x\}$.
- FL based on deriving empty clause from assumption and BCP.

FL for QBF:

- Def.: failed literal x for PCNF ψ: if $\psi \equiv \psi \wedge \{\neg x\}$.
- Problem: BCP-based approach like for SAT is unsound due to $\exists/\forall$ prefix.

Example

$\psi := \forall x \exists y. \{x, \neg y\}, \{\neg x, y\}$. We have $\emptyset \in BCP(\psi, y)$ but $\psi \not\equiv \psi \wedge \{\neg y\}$.

Our Work:

- Two orthogonal FL approaches for QBF.
- Soundness established by abstraction and Q-resolution.

Motivation

Failed Literal Detection (FL) for SAT:

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Abstraction-Based FL

Problem: $BCP(\psi, x)$ with assumption x for FL on PCNF ψ is unsound.

Definition (Quantifier Abstraction)

For $\psi := Q_1 S_1 \ldots Q_{i-1} S_{i-1} Q_i S_i \ldots \ldots Q_n S_n.\ \phi$, the quantifier abstraction of ψ with respect to S_i is $Abs(\psi, i) := \exists (S_1 \cup \ldots \cup S_{i-1}) Q_i S_i \ldots Q_n S_n.\ \phi$.

Idea: carry out BCP on abstraction of ψ.

- If $x \in S_i$ then treat all variables smaller than x as existentially quantified.
- Example: $Abs(\exists x \forall y \exists z.\ \phi, 3) = \exists x \exists y \exists z.\ \phi$.
- Overapproximation: if $m \models \psi$ then $m \models Abs(\psi, i)$.

Theorem

Given PCNF $\psi := Q_1 S_1 \ldots Q_n S_n.\ \phi$ *and literal* x *where* $v(x) \in S_i$. *If* $\emptyset \in BCP(Abs(\psi, i), x)$ *then* $\psi \equiv \psi \wedge \{\neg x\}$.

Practical Application:

- FL using BCP on abstraction is sound and runs in polynomial-time.

BCP-Guided Q-Resolution (1/2)

Definition (Q-resolution)

Let C_1, C_2 be clauses with $v \in C_1, \neg v \in C_2$ and $q(v) = \exists$ [BKF95].

1. $C_1 \otimes C_2 := (UR(C_1) \cup UR(C_2)) \setminus \{v, \neg v\}$.
2. If $\{x, \neg x\} \subseteq C_1 \otimes C_2$ (tautology) then no Q-resolvent exists.
3. Otherwise, Q-resolvent $C := UR(C_1 \otimes C_2)$ of C_1 and C_2 on v: $\{C_1, C_2\} \vdash^* C$.

Q-Resolution: combination of propositional resolution and UR.

- For PCNF ψ, clause C: if $\psi \vdash^* C$ then $\psi \equiv \psi \wedge C$.

Idea: (heuristically) validate $\emptyset \in BCP(\psi, x)$ on *original* PCNF.

- Try to derive the negated assumption $\{\neg x\}$ by Q-resolution.
- Resolution candidates are selected from clauses “touched” by BCP.
- Like conflict-driven clause learning (CDCL) in search-based solvers.

Novel Approach: BCP-Guided Q-Resolution (2/2)

Corollary

Given PCNF $\psi := Q_1 S_1 \ldots Q_n S_n.\ \phi$ *and literal* x *where* $v(x) \in S_i$. *If* $\emptyset \in BCP(\psi, x)$ *and* $\psi \vdash^* \{\neg x\}$ *then* $\psi \equiv \psi \wedge \{\neg x\}$.

Example

$\psi := \exists e_1, e_2 \forall a_3 \exists e_4, e_5.\ \{a_3, e_5\}, \{\neg e_2, e_4\}, \{\neg e_1, e_4\}, \{e_1, e_2, \neg e_5\}$.
With assumption $\neg e_4$ we get $\emptyset \in BCP(\psi, \neg e_4)$ since $\{\neg e_1\}$, $\{\neg e_2\}$ and $\{\neg e_5\}$ become unit. Finally $\{a_3, e_5\}$ is empty by UR.
The negated assumption $\{e_4\}$ is then derived by resolving clauses in reverse-chronological order as they were affected by assignments:
$(\{a_3, e_5\}, \{e_1, e_2, \neg e_5\}) \vdash \{e_1, e_2\}$, $(\{e_1, e_2\}, \{\neg e_2, e_4\}) \vdash \{e_1, e_4\}$,
$(\{e_1, e_4\}, \{\neg e_1, e_4\}) \vdash \{e_4\}$.

Practical Application:

- Advantage: original prefix allows full propagation power in BCP.
- BCP-based selection of resolution candidates is only a heuristic.

Comparing FL Approaches

Proposition

Abstraction-based FL and BCP-guided Q-resolution are orthogonal to each other with respect to detecting necessary assignments.

Consequences:

- There are PCNFs where one approach can detect a necessary assignment the other one cannot.
- No approach can detect all necessary assignments.
- Crucial observation: Q-resolution for CDCL is not optimal (see below)!
- (How) Can we apply quantifier abstraction for clause learning?

Example

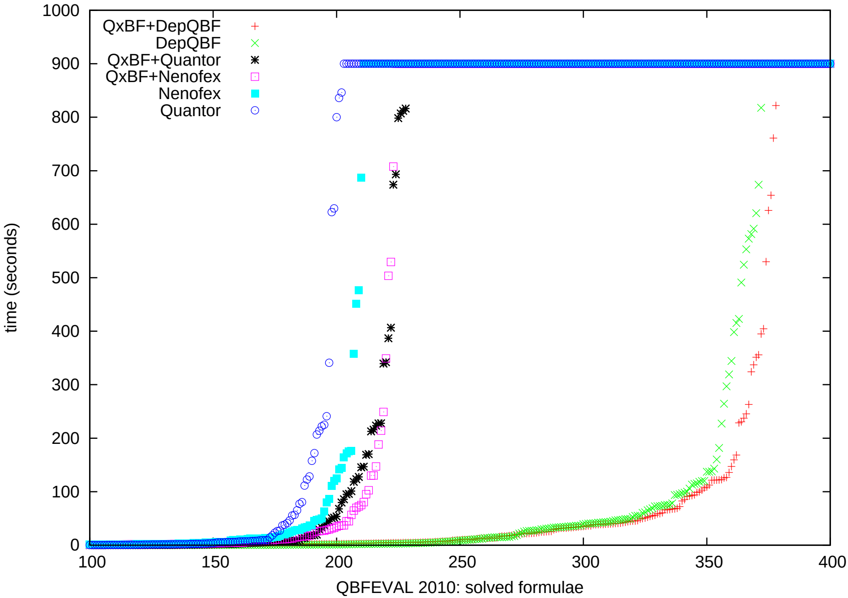
$\psi := \forall a_1 \exists e_2, e_3 \forall a_4 \exists e_5.\ \{a_1, e_2\}, \{\neg a_1, e_3\}, \{e_3, \neg e_5\}, \{a_1, e_2, \neg e_3\}, \{\neg e_2, a_4, e_5\}$. We have $\emptyset \in BCP(Abs(\psi, 2), \neg e_3)$ but $\psi \not\vdash^* \{e_3\}$: assignment $\{e_3\} \mapsto$ *true* is necessary but Q-resolution can *not* derive clause $\{e_3\}$.

Tool "QxBF": FL-based preprocessor operating in rounds.
SAT-Based FL: using SAT solver to detect necessary assignments.

QBFEVAL'10: 568 formulae					
Preprocessing	*Solver*	*Solved*	*Time (Preproc.)*	*SAT*	*UNSAT*
SAT	DepQBF	379	322.31 (7.17)	167	212
QRES+SAT		378	322.83 (6.22)	167	211
ABS+SAT		378	323.19 (7.21)	167	211
ABS		375	327.64 (3.33)	168	207
QRES		374	327.63 (1.83)	167	207
None		372	334.60 (—)	166	206
ABS+SAT	Quantor	229	553.65 (7.21)	112	117
	Nenofex	224	553.37 (7.21)	104	120
none		211	573.65 (—)	103	108
	Quantor	203	590.15 (—)	99	104
ABS+SAT	squolem	154	658.28 (7.21)	63	91
None		124	708.80 (—)	53	71

Table: Solver performance with(out) time-limited failed literal preprocessing. Search-based solver DepQBF, elimination-based solvers Quantor, squolem, Nenofex. No preprocessing ("none"), SAT-based FL ("SAT"), abstraction-based FL ("ABS") and BCP-guided Q-resolution ("QRES").

FL Times Plot



Part 3: Quantified Blocked Clause Elimination (QBCE)

Blocked Clause Elimination (BCE) for SAT [JBH10]

- Allows CNF-level simplifications after circuit-to-CNF transformation.
- At least as effective as many circuit-level preprocessing techniques.
- Simulates pure literal rule, Plaisted-Greenbaum encoding, ...

Quantified Blocked Clause Elimination (QBCE) for QBF

- Paper submitted to CADE'11 (joint work with Armin Biere, Martina Seidl).
- Generalizes BCE to QBF: minor but crucial adaption of BCE definition.
- Implementation: tool "bloqqer" combines QBCE and extensions with variable elimination, self-subsuming resolution, subsumption,...

Definition of QBCE: based checking possible Q-resolvents.

Definition

Let C_1, C_2 be clauses with $v \in C_1, \neg v \in C_2$ and $q(v) = \exists$.

1. Tentative resolvent: $C_1 \otimes C_2 := (UR(C_1) \cup UR(C_2)) \setminus \{v, \neg v\}$.

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QBCE Definition

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Given PCNF $Q_1 S_1 \ldots Q_n S_n.\ (\phi \wedge C)$. Clause C is *quantified blocked* if it contains a quantified blocking literal.

Then $Q_1 S_1 \ldots Q_n S_n.\ (\phi \wedge C) \overset{sat}{\equiv} Q_1 S_1 \ldots Q_n S_n.\ \phi$.

$C_1 \in Occs(l)$ blocked?	$C_2 \in Occs(\neg l)$	$C_1 \otimes C_2$
$(x_1 \vee x_2 \vee \ldots \vee x_n \vee \ldots \vee l \vee \ldots)$	$(\ldots \neg x_1 \vee \ldots \vee \neg l \vee \ldots)$	$\{x_1, \neg x_1\} \in C_1 \otimes C_2$
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Example

All clauses blocked: $\forall x \exists y ((x \vee \neg y) \wedge (\neg x \vee y))$.
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Then $Q_1 S_1 \ldots Q_n S_n.\ (\phi \wedge C) \overset{sat}{\equiv} Q_1 S_1 \ldots Q_n S_n.\ \phi$.

$C_1 \in Occs(l)$ blocked?	$C_2 \in Occs(\neg l)$	$C_1 \otimes C_2$
$(x_1 \vee x_2 \vee \ldots \vee x_n \vee \ldots \vee l \vee \ldots)$	$(\ldots \neg x_1 \vee \ldots \vee \neg l \vee \ldots)$	$\{x_1, \neg x_1\} \in C_1 \otimes C_2$
	$(\ldots \neg x_2 \vee \ldots \vee \neg l \vee \ldots)$	$\{x_2, \neg x_2\} \in C_1 \otimes C_2$
	$\ldots$	
	$(\ldots \neg x_n \vee \ldots \vee \neg l \vee \ldots)$	$\{x_n, \neg x_n\} \in C_1 \otimes C_2$

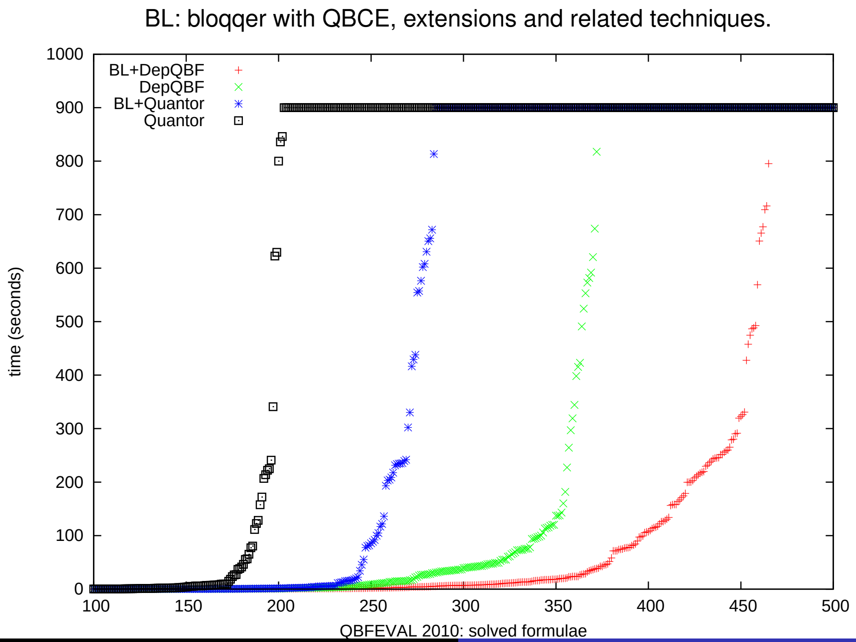
Example

All clauses blocked: $\forall x \exists y((x \vee \neg y) \wedge (\neg x \vee y))$.
No clause blocked: $\exists x \forall y((x \vee \neg y) \wedge (\neg x \vee y))$.

Table: Bloqqer (QBCE, extensions and related techniques) combined with search- (DepQBF, QuBE) and elimination-based (Nenofex, Quantor) solvers.

QBFEVAL'10: 568 formulae								
		# formulas			runtime (sec)		
	preprocessor	SOLVED	SAT	UNSAT	$\sum (10^3)$	AVG	MEDIAN
DepQBF	bloqqer	467	224	243	112	198	5
	no preprocessing	373	167	206	189	332	26
QuBE	bloqqer	444	200	244	139	246	5
	no preprocessing	332	135	197	242	426	258
Quantor	bloqqer	288	145	143	266	468	34
	no preprocessing	206	100	106	333	587	38
Nenofex	bloqqer	268	132	136	276	487	23
	no preprocessing	221	107	114	319	561	113

QBCE Times Plot



Preprocessing QBF:

- Positive effects on elimination- and search-based QBF solvers.

Failed Literal Detection (FL):

- Detecting a subset of necessary assignments.
- Exponential reduction of search-space.
- Soundness by abstraction and Q-resolution.
- Orthogonality: current CDCL approaches in QBF are not optimal.

Quantified Blocked Clause Elimination (QBCE):

- Generalizes BCE for SAT to QBF.
- Best performance when combined with variable elimination,. . .

Work in Progress:

- Papers submitted to SAT'11 (FL) and CADE'11 (QBCE).
- Source code of our preprocessors will be published.
- Dynamic applications of FL and QBCE.

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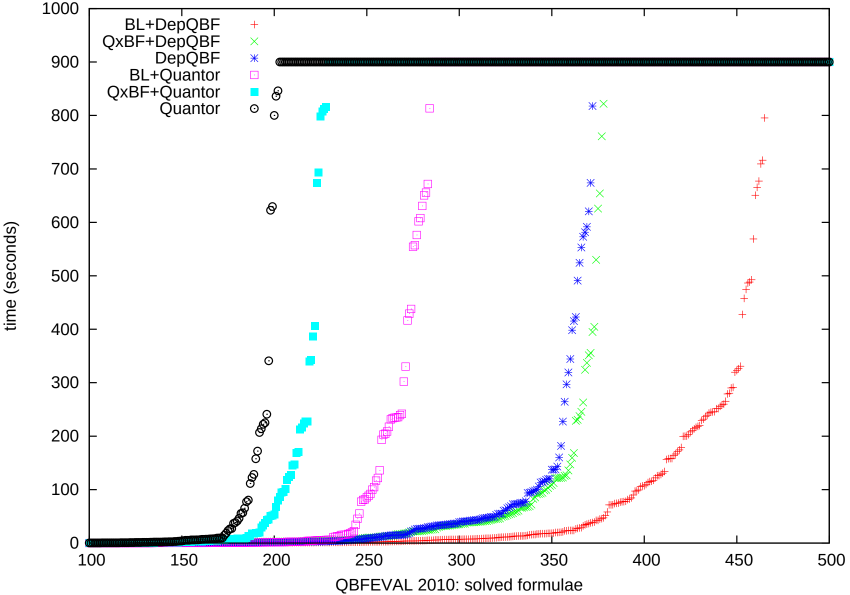

1000
900
800
700
600
500
400
300
200
100
0
time (seconds)
100
150
200
250
300
350
400
450
500
QBFEVAL 2010: solved formulae
BL+DepQBF
QxBF+DepQBF
DepQBF
BL+Quantor
QxBF+Quantor
Quantor

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