

# Conflict-driven ASP Solving with External Source Access

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September 05, 2012

# Motivation

## HEX-Programs

- Extend ASP by **external sources**
- Current algorithm based on a **translation to ASP**
- **Scalability problems**

## Contribution

- New **genuine algorithms**
- Based on **conflict-driven algorithms**
- Much **better scalability**

# Outline

- 1 Introduction
- 2 Algorithms with External Behavior Learning
- 3 Nogood Generation for External Behavior Learning
- 4 Implementation and Evaluation
- 5 Conclusion

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# HEX-Programs

HEX-programs extend ordinary ASP programs by **external sources**

## Definition (HEX-programs)

A **HEX-program** consists of rules of form

$$a_1 \vee \dots \vee a_n \leftarrow b_1, \dots, b_m, \text{not } b_{m+1}, \dots, \text{not } b_n,$$

with classical literals  $a_i$ , and classical literals or an external atoms  $b_j$ .

## Definition (External Atoms)

An **external atom** is of the form

$$\&p[q_1, \dots, q_k](t_1, \dots, t_l),$$

$p$  ... external predicate name

$q_i$  ... predicate names or constants

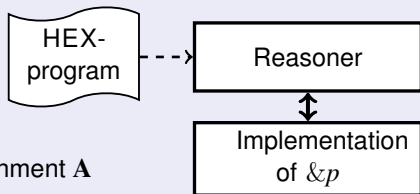
$t_j$  ... terms

Semantics:

$1 + k + l$ -ary Boolean **oracle function**  $f_{\&p}$ :

$\&p[q_1, \dots, q_k](t_1, \dots, t_l)$  is true under assignment  $\mathbf{A}$

iff  $f_{\&p}(\mathbf{A}, q_1, \dots, q_k, t_1, \dots, t_l) = 1$ .



# Examples

## *&rdf*

### The *&rdf* External Atom

- Input: URL
- Output: Set of triplets from RDF file

External knowledge base is a set of RDF files on the web:

$$addr(\text{http://.../data1.rdf}).$$

$$addr(\text{http://.../data2.rdf}).$$

$$bel(X, Y) \leftarrow addr(U), \&rdf[U](X, Y, Z).$$

# Examples

## *&rdf*

### The *&rdf* External Atom

- Input: URL
- Output: Set of triplets from RDF file

External knowledge base is a set of RDF files on the web:

$$\begin{aligned} &addr(\text{http://.../data1.rdf}). \\ &addr(\text{http://.../data2.rdf}). \\ &bel(X, Y) \leftarrow addr(U), \&rdf[U](X, Y, Z). \end{aligned}$$

## *&diff*

$\&diff[p, q](X)$ : all elements  $X$ , which are in the extension of  $p$  but not of  $q$ :

$$\begin{aligned} dom(X) &\leftarrow \#int(X). \\ nsel(X) &\leftarrow dom(X), \&diff[dom, sel](X). \\ sel(X) &\leftarrow dom(X), \&diff[dom, nsel](X). \\ &\leftarrow sel(X1), sel(X2), sel(X3), X1 \neq X2, X1 \neq X3, X2 \neq X3. \end{aligned}$$

# Current Evaluation Method

## Evaluating Program $\Pi$

- 1 Replace external atoms  $\&g[\vec{p}](\vec{c})$  by **ordinary ones**  $e_{\&g[\vec{p}]}(\vec{c})$  and **guess** their values  $\rightarrow$  **guessing program**  $\hat{\Pi}$
- 2 For each candidate, **check** if the truth values coincide with external sources
- 3 Check if  $\mathbf{A}$  is subset-minimal under all compatible sets

## Definition (Compatible Set)

A **compatible set** of a program  $\Pi$  is an assignment  $\mathbf{A}$

- (i) which is an answer set [Gelfond and Lifschitz, 1991] of  $\hat{\Pi}$ , and
- (ii)  $f_{\&g}(\mathbf{A}, \vec{p}, \vec{c}) = 1$  iff  $\mathbf{T}e_{\&g[\vec{p}]}(\vec{c}) \in \mathbf{A}$  for all external atoms  $\&g[\vec{p}](\vec{c})$  in  $\Pi$

## Definition (Answer Set)

An **(DLVHEX) answer set** of  $\Pi$  is any set  $S \subseteq \{\mathbf{T}a \mid a \in A(\Pi)\}$  such that

- (i)  $S = \{\mathbf{T}a \mid a \in A(\Pi)\} \cap \mathbf{A}$  for some compatible set  $\mathbf{A}$  of  $\Pi$  and
- (ii)  $S \not\subseteq \{\mathbf{T}a \mid a \in A(\Pi)\} \cap \mathbf{A}$  for every compatible set  $\mathbf{A}$  of  $\Pi$ .



# Current Evaluation Method

## Translation Approach

HEX-Program  $\Pi$ :

$$p(c_1). \text{dom}(c_1). \text{dom}(c_2). \text{dom}(c_3). \\ p(X) \leftarrow \text{dom}(X), \&\text{empty}[p](X).$$

Guessing program  $\hat{\Pi}$ :

$$p(c_1). \text{dom}(c_1). \text{dom}(c_2). \text{dom}(c_3). \\ p(X) \leftarrow \text{dom}(X), e_{\&\text{empty}[p]}(X). \\ e_{\&\text{empty}[p]}(X) \vee \neg e_{\&\text{empty}[p]}(X) \leftarrow \text{dom}(X).$$

8 candidates, e.g.:

$$\{\mathbf{T}p(c_1), \mathbf{T}p(c_2), \mathbf{T}\text{dom}(c_1), \mathbf{T}\text{dom}(c_2), \mathbf{T}\text{dom}(c_3), \\ \mathbf{F}e_{\&\text{empty}[p]}(c_1), \mathbf{T}e_{\&\text{empty}[p]}(c_2), \mathbf{F}e_{\&\text{empty}[p]}(c_3)\}$$

Compatibility check: **passed**  $\Rightarrow$  **compatible set**

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# Novel Algorithm

## Idea

- Use a **conflict-driven** (disjunctive) ASP solver [Drescher et al., 2008]
- Program  $\hat{\Pi}$  is represented as a **set of nogoods**

# Novel Algorithm

## Idea

- Use a **conflict-driven** (disjunctive) ASP solver [Drescher et al., 2008]
- Program  $\hat{\Pi}$  is represented as a **set of nogoods**
- Introduce **additional nogoods** to describe external sources

## Definition (Assignment)

An **assignment** is a consistent set of *signed literals*  $\mathbf{T}a$  or  $\mathbf{F}a$ , where  $a$  is an atom.

## Definition (Nogoods)

A **nogood** is a set of *signed literals*.

## Definition (Solution to Nogoods)

An assignment  $\mathbf{A}$  is a **solution** to a set of nogoods  $\Delta$  iff  $\delta \not\subseteq \mathbf{A}$  for all  $\delta \in \Delta$ .

# Basic Definitions

## Definition (Correct Nogoods)

A nogood  $\delta$  is **correct wrt. program  $\Pi$** , if all compatible sets of  $\Pi$  are solutions to  $\delta$ .

## Definition (Learning Function)

A **learning function** for  $\Pi$  is a mapping  $\Lambda : \mathcal{E} \times 2^{\mathcal{D}} \mapsto 2^{2^{\mathcal{D}}}$

$\mathcal{E}$  ... set of all external predicates with input list in  $\Pi$

$\mathcal{D}$  ... set of all signed literals

## Definition (Correct Learning Functions)

A learning function  $\Lambda$  is **correct** for a program  $\Pi$ , if and only if all  $d \in \Lambda(\&g[\vec{p}], \mathbf{A})$  are correct for  $\Pi$ , for all  $\&g[\vec{p}]$  in  $\mathcal{E}$  and  $\mathbf{A} \in 2^{\mathcal{D}}$ .

# Algorithms

## Algorithm: HEX-Eval

**Input:** A HEX-program  $\Pi$

**Output:** All answer sets of  $\Pi$

$\hat{\Pi} \leftarrow \Pi$  with all external atoms  $\&g[\bar{p}](\vec{c})$  replaced by  $e_{\&g[\bar{p}]}(\vec{c})$

Add guessing rules for all replacement atoms to  $\hat{\Pi}$

$\nabla \leftarrow \emptyset$  /\* set of dynamic nogoods \*/

$\Gamma \leftarrow \emptyset$  /\* set of all compatible sets \*/

**while**  $C \neq \perp$  **do**

$C \leftarrow \perp$

*inconsistent*  $\leftarrow$  *false*

**while**  $C = \perp$  and *inconsistent* = *false do*

$A \leftarrow$  HEX-CDNL( $\Pi, \hat{\Pi}, \nabla$ )

**if**  $A = \perp$  **then**

*inconsistent*  $\leftarrow$  *true*

**else**

*compatible*  $\leftarrow$  *true*

**for all external atoms**  $\&g[\bar{p}]$  **in**  $\Pi$  **do**

Evaluate  $\&g[\bar{p}]$  under  $A$

$\nabla \leftarrow \nabla \cup \Lambda(\&g[\bar{p}], A)$

Let  $A_{\&g[\bar{p}]}(\vec{c}) = 1 \Leftrightarrow T_{e_{\&g[\bar{p}]}}(\vec{c}) \in A$

**if**  $\exists \vec{c}: f_{\&g}(\vec{c}) \neq A_{\&g[\bar{p}]}(\vec{c})$  **then**

Add  $A$  to  $\nabla$

Set *compatible*  $\leftarrow$  *false*

**end**

**if** *compatible* **then**  $C \leftarrow A$

**end**

**if** *inconsistent* = *false* **then**

/\*  $C$  is a compatible set of  $\Pi$  \*/

$\nabla \leftarrow \nabla \cup \{C\}$  and  $\Gamma \leftarrow \Gamma \cup \{C\}$

Output  $\{\{T_a \in A \mid a \in A(\Pi)\} \mid A \in \Gamma\}$  which are subset-minimal

## Algorithm: HEX-CDNL

**Input:** A program  $\Pi$ , its guessing program  $\hat{\Pi}$ , a set of correct nogoods  $\nabla$  of  $\Pi$

**Output:** An answer set of  $\hat{\Pi}$  (candidate for a compatible set of  $\Pi$ ) which is a solution to all nogoods  $d \in \nabla$ , or  $\perp$  if none exists

$A \leftarrow \emptyset$  /\* assignment over  $A(\hat{\Pi}) \cup BA(\hat{\Pi}) \cup BA(sh(\hat{\Pi}))$  \*/

$dl \leftarrow 0$  /\* decision level \*/

**while** *true do*

$(A, \nabla) \leftarrow$  Propagation( $\hat{\Pi}, \nabla, A$ )

**if**  $\delta \subseteq A$  for some  $\delta \in \Delta_{\hat{\Pi}} \cup \Theta_{sh(\hat{\Pi})} \cup \nabla$  **then**

**if**  $dl = 0$  **then** **return**  $\perp$

$(\epsilon, k) \leftarrow$  Analysis( $\delta, \hat{\Pi}, \nabla, A$ )

$\nabla \leftarrow \nabla \cup \{\epsilon\}$

$A \leftarrow A \setminus \{\sigma \in A \mid k < dl(\sigma)\}$

$dl \leftarrow k$

**else if**  $A^T \cup A^F = A(\hat{\Pi}) \cup BA(\hat{\Pi}) \cup BA(sh(\hat{\Pi}))$  **then**

$U \leftarrow$  UnfoundedSet( $\hat{\Pi}, A$ )

**if**  $U \neq \emptyset$  **then**

let  $\delta \in \lambda_{\hat{\Pi}}(U)$  such that  $\delta \subseteq A$

**if**  $\{\sigma \in \delta \mid 0 < dl(\sigma)\} = \emptyset$  **then** **return**  $\perp$

$(\epsilon, k) \leftarrow$  Analysis( $\delta, \hat{\Pi}, \nabla, A$ )

$\nabla \leftarrow \nabla \cup \{\epsilon\}$

$A \leftarrow A \setminus \{\sigma \in A \mid k < dl(\sigma)\}$

$dl \leftarrow k$

**else**

**return**  $A^T \cap A(\hat{\Pi})$

**else if** *Heuristic decides to evaluate*  $\&g[\bar{p}]$  **then**

Evaluate  $\&g[\bar{p}]$  under  $A$  and set  $\nabla \leftarrow \nabla \cup \Lambda(\&g[\bar{p}], A)$

**else**

$\sigma \leftarrow$  Select( $\hat{\Pi}, \nabla, A$ )

$dl \leftarrow dl + 1$

$A \leftarrow A \circ (\sigma)$

# Algorithms

Restricting to learning functions that are correct for  $\Pi$ , the following results hold.

## Proposition

If for input  $\Pi$ ,  $\hat{\Pi}$  and  $\nabla$ , HEX-CDNL returns

- (i) *an interpretation  $\mathbf{A}$ , then  $\mathbf{A}$  is an answer set of  $\hat{\Pi}$  and a solution to  $\nabla$ ;*
- (ii)  *$\perp$ , then  $\Pi$  has no compatible set that is a solution to  $\nabla$ .*

## Proposition

HEX-Eval computes all answer sets of  $\Pi$ .

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# Concrete Learning Functions

**Idea:** learn that input implies output

## Definition

The learning function for a general external predicate with input list  $\&g[\vec{p}]$  in program  $\Pi$  under assignment  $\mathbf{A}$  is defined as

$$\Lambda_g(\&g[\vec{p}], \mathbf{A}) = \{ \mathbf{A}|_{\vec{p}} \cup \{ \mathbf{F}e_{\&g[\vec{p}]}(\vec{c}) \} \mid (\vec{c}) \in \text{ext}(\&g[\vec{p}], \mathbf{A}) \}$$

## Example

$\&diff[p, q](X)$  with  $\text{ext}(p, \mathbf{A}) = \{a, b\}$ ,  $\text{ext}(q, \mathbf{A}) = \{a, c\}$

Learn:  $\{ \mathbf{T}p(a), \mathbf{T}p(b), \mathbf{F}p(c), \mathbf{T}q(a), \mathbf{F}q(b), \mathbf{T}q(c), \mathbf{F}e_{\&diff[p, q]}(b) \}$

## Lemma

*For all assignments  $\mathbf{A}$ , the nogoods  $\Lambda_g(\&g[\vec{p}], \mathbf{A})$  are correct wrt.  $\Pi$ .*

# Concrete Learning Functions

**Idea:** learn that parts of the input imply output

## Definition

The learning function for an external predicate  $\&g$  with input list  $\vec{p}$  in program  $\Pi$  under assignment  $\mathbf{A}$ , such that  $\&g$  is monotonic in  $\vec{p}_m \subseteq \vec{p}$ , is defined as

$$\Lambda_m(\&g[\vec{p}], \mathbf{A}) = \left\{ \{ \mathbf{T}a \in \mathbf{A} \mid_{\vec{p}_m} \} \cup \mathbf{A} \mid_{\vec{p}_n} \cup \{ \mathbf{F}e_{\&g[\vec{p}]}(\vec{c}) \} \mid (\vec{c}) \in \text{ext}(\&g[\vec{p}], \mathbf{A}) \right\}$$

## Example

$\&diff[p, q](X)$  with  $\text{ext}(p, \mathbf{A}) = \{a, b\}$ ,  $\text{ext}(q, \mathbf{A}) = \{a, c\}$ , monotonic in  $p$

Learn:  $\{ \mathbf{T}p(a), \mathbf{T}p(b), \mathbf{T}q(a), \mathbf{F}q(b), \mathbf{T}q(c), \mathbf{F}e_{\&diff[p, q]}(b) \}$

## Lemma

For all assignments  $\mathbf{A}$ , the nogoods  $\Lambda_m(\&g[\vec{p}], \mathbf{A})$  are correct wrt.  $\Pi$ .

# Concrete Learning Functions

**Idea:** multiple output tuples exclude each other

## Definition

The learning function for a functional external predicate  $\&g$  with input list  $\vec{p}$  in program  $\Pi$  under assignment  $\mathbf{A}$  is defined as

$$\Lambda_f(\&g[\vec{p}], \mathbf{A}) = \left\{ \{ \mathbf{T}e_{\&g[\vec{p}]}(\vec{c}), \mathbf{T}e_{\&g[\vec{p}]}(\vec{c}') \} \mid \vec{c} \neq \vec{c}' \right\}$$

## Example

$\&concat[ab, c](X)$

Learn:  $\{ \mathbf{T}e_{\&concat[ab, c]}(abc), \mathbf{T}e_{\&concat[ab, c]}(ab) \}$

## Lemma

*For all assignments  $\mathbf{A}$ , if  $\&g$  is functional, the nogoods  $\Lambda_f(\&g[\vec{p}], \mathbf{A})$  are correct wrt.  $\Pi$ .*

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# Implementation

## Implementation

- Prototype implementation: DLVHEX
- Written in C++
- External sources loaded via plugin interface

## Technology

- Basis: Gringo and CLASP
- External Behavior Learning exploits CLASP's SMT interface
- Alternatively: self-made grounder and solver built from scratch

# Benchmark Results

## Set Partitioning

| #  | all models |        |       | first model |       |       |
|----|------------|--------|-------|-------------|-------|-------|
|    | DLV        | CLASP  | CLASP | DLV         | CLASP | CLASP |
|    | w/o EBL    | w EBL  | w EBL | w/o EBL     | w EBL | w EBL |
| 1  | 0.07       | 0.08   | 0.07  | 0.08        | 0.07  | 0.07  |
| 5  | 0.20       | 0.16   | 0.10  | 0.08        | 0.08  | 0.07  |
| 10 | 12.98      | 9.56   | 0.17  | 0.56        | 0.28  | 0.07  |
| 11 | 38.51      | 21.73  | 0.19  | 0.93        | 0.63  | 0.08  |
| 12 | 89.46      | 49.51  | 0.19  | 1.69        | 1.13  | 0.08  |
| 13 | 218.49     | 111.37 | 0.20  | 3.53        | 2.31  | 0.10  |
| 14 | —          | 262.67 | 0.28  | 8.76        | 3.69  | 0.10  |
| ⋮  |            |        | ⋮     | ⋮           | ⋮     | ⋮     |
| ⋮  | —          | —      | ⋮     | ⋮           | ⋮     | ⋮     |
| 18 | —          | —      | 0.45  | 128.79      | 62.58 | 0.12  |
| 19 | —          | —      | 0.42  | —           | 95.39 | 0.10  |
| 20 | —          | —      | 0.54  | —           | 91.16 | 0.11  |

## Bird-Penguin

| #  | DLV     | CLASP  | CLASP |
|----|---------|--------|-------|
|    | w/o EBL | w EBL  | w EBL |
| 1  | 0.50    | 0.15   | 0.14  |
| 5  | 1.90    | 1.98   | 0.59  |
| 6  | 4.02    | 4.28   | 0.25  |
| 7  | 8.32    | 7.95   | 0.60  |
| 8  | 16.11   | 16.39  | 0.29  |
| 9  | 33.29   | 34.35  | 0.35  |
| 10 | 83.75   | 94.62  | 0.42  |
| 11 | 229.20  | 230.75 | 4.45  |
| 12 | —       | —      | 1.10  |
| ⋮  |         |        | ⋮     |
| ⋮  | —       | —      | ⋮     |
| 20 | —       | —      | 2.70  |

### HEX — Program :

$$\begin{aligned} \text{birds}(X) &\leftarrow DL[\text{Bird}](X). \\ \text{flies}(X) &\leftarrow \text{birds}(X), \text{not } \text{neg\_flies}(X). \\ \text{neg\_flies}(X) &\leftarrow \text{birds}(X), DL[\text{Flier} \sqcup \text{flies}; \neg\text{Flier}](X). \end{aligned}$$

### Ontology :

$$\begin{aligned} \text{Flier} &\sqsubseteq \neg\text{NonFlier} \\ \text{Penguin} &\sqsubseteq \text{Bird} \\ \text{Penguin} &\sqsubseteq \text{NonFlier} \end{aligned}$$


# Benchmark Results

## Wine Classification

“A wine is white by default, unless it is derivable that it is red”

| Inst.  | concept completion |       | speedup |      |
|--------|--------------------|-------|---------|------|
|        | w/o EBL            | w EBL | max     | avg  |
| wine_0 | 25                 | 31    | 33.02   | 6.93 |
| wine_1 | 16                 | 25    | 16.05   | 5.78 |
| wine_2 | 14                 | 22    | 11.82   | 4.27 |
| wine_3 | 4                  | 17    | 10.09   | 4.02 |
| wine_4 | 4                  | 17    | 6.83    | 2.87 |
| wine_5 | 4                  | 16    | 5.22    | 2.34 |
| wine_6 | 4                  | 13    | 2.83    | 1.52 |
| wine_7 | 4                  | 12    | 1.81    | 1.14 |
| wine_8 | 4                  | 4     | 1.88    | 1.08 |



## Explaining Inconsistency in Multi-context Systems

| #contexts | DLV  | CLASP   |       |
|-----------|------|---------|-------|
|           |      | w/o EBL | w EBL |
| 3         | 0.07 | 0.05    | 0.04  |
| 4         | 1.04 | 0.68    | 0.14  |
| 5         | 0.23 | 0.15    | 0.05  |
| 6         | 2.63 | 1.44    | 0.12  |
| 7         | 8.71 | 4.39    | 0.17  |

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# Conclusion

## External Behavior Learning (EBL)

- Provide novel **genuine algorithms** for HEX-program evaluation
- Use **customizable learning functions**
- Learn additional nogoods from external source evaluations
- **Uninformed** vs. **informed** learning

## Implementation and Evaluation

- Prototype implementation based on Gringo and CLASP
- Experiments show significant **improvements by EBL**

## Future Work

- Identify **further properties** for informed learning
- **Language** for writing user-defined learning functions

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