A Tableau Algorithm for Handling Inconsistency in OWL*

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Abstract. In Semantic Web, the knowledge sources usually contain inconsistency because they are constantly changing and from different view points. As is well known, as based on the description logic, OWL is lack of the ability of tolerating inconsistent or incomplete data. Recently, the research in handling inconsistency in OWL becomes more and more important. In this paper, we present a paraconsistent OWL called quasi-classical OWL to handle inconsistency with holding important inference rules such as modus tollens, modus ponens, and disjunctive syllogism. We propose a terminable, sound and complete tableau algorithm to implement paraconsistent reasoning in quasi-classical OWL. In comparison with other approaches to handle inconsistency in OWL, our approach enhances the ability of reasoning by integrating paraconsistent reasoning with important classical inference rules.

1 Introduction

In recent years, the problem of inconsistency handling in OWL is attracting a lot of attention in logics and Semantic Web. Many reasons cause the occurrence of inconsistency such as modeling errors, migration from other formalisms, merging ontologies, and ontology evolution [1]. In practical reasoning, it is common to have "too much" information about some situation. In other words, it is common for there to be classically inconsistent information in practical reasoning ontologies [2]. According the fact *ex contradictione quodlibet* in classical logic, if ontologies contain inconsistencies then the classical entailment in logics is explosive. That is to say, any formula is a logical consequence of a contradiction. Therefore, conclusions drawn from an inconsistent knowledge base may be completely meaningless [3]. This is particularly important if the full power of logic-based approaches like the Web Ontology Language (short OWL) [4] shall be employed, as classical logic breaks down in the presence of inconsistent knowledge. Not surprisingly, the study of handling inconsistency in OWL becomes more and more important.

There are serval approaches to handling inconsistency in OWL, which can be generally divided into two fundamentally different approaches. The first is based on the assumption that inconsistencies indicate erroneous data which are to be repaired in order to obtain a consistent knowledge base, e.g. by selecting consistent subsets for the

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reasoning process [3,5,6]. Another approach, called paraconsistent approach, is not to simply avoid the inconsistency but to apply a non-standard reasoning method to obtain meaningful answers [7,8,9,10,11]. In the paraconsistent approach, inconsistencies are treated as a natural phenomenon in realistic data and are tolerated in reasoning. Compared with the former, the latter acknowledges and distinguishes the different epistemic statuses between "the assertion is true" and "the assertion is true with conflict". So far, the main idea of paraconsistent approach of handling inconsistency in OWL is borrowing Belnaps four-valued semantics [12] for OWL. The most prominent of them are based on the use of additional truth values standing for $underdefined$ (i.e. neither true nor false) and $overdefined$ (or $contradictory$, i.e, both true and false). Four-valued semantics proves useful for measuring inconsistency of ontologies [13], which can provide context information for facilitating inconsistency handling. To a certain extent four-valued semantics handles inconsistency, however the capability of reasoning is weaker than classical OWL because four-valued OWL doesn't hold three basic inference rules such as modus ponens, disjunctive syllogism, modus tollens or intuitive equivalences. In [8,9], a total negation is introduced to strengthen the capability of paraconsistent reasoning in four-valued OWL by applying resolution principles. However, four-valued OWL with the total negation don't hold intuitive equivalences. These shortcomings are inherent limitations of four-valued semantics in reasoning.

Naturally, we expect that there is a paraconsistent logic which has the same capability of reasoning in classical logics to handle inconsistency in OWL. However, it is still an open problem now. Motivation of this paper is to find a paraconsistent approach to OWL with three basic inference rules and intuitive equivalences.

OWL is based on description logics and the description logic $\mathcal{ALCNQ}$ is considered to be the most foundational one and comprises a large fragment of OWL. In this paper, based on [14], we study a description logic $\mathcal{ALCNQ}$ version of paraconsistent logic, called quasi-classical (or QC) $\mathcal{ALCNQ}$, which is an extension of quasi-classical logic [15,16] and a paraconsistent version of $\mathcal{ALCNQ}$. We contribute to the inconsistency handling for $\mathcal{ALCNQ}$ in terms of the QC semantics in the following aspects.

- A new semantics called QC semantics including two semantics "QC weak semantics" and "QC strong semantics" is introduced to description logic $\mathcal{ALCNQ}$ in this paper. QC weak semantics inherits the characters from four-valued semantics in order to reason paraconsistently, while QC strong semantics is introduced to strengthen the capability of reasoning in ontologies. Compared to QC weak semantics, QC strong semantics refines the disjunction of concepts in order to hold three basic inference rules. Moreover, concept subsumption redefined in QC strong semantics is different from four-valued description logic in order to hold intuitive equivalences, i.e., $C \sqsubseteq D$ iff $\neg C \sqcup D(a)$ for any individual a occurring in ontologies.
- A QC entailment (written by "$\models_Q$") between an ontology and an axiom is presented by applying QC semantics in this paper. "$\models_Q$" is a nontrivial entailment, i.e., $\{C(a), \neg C(a)\} \not\models_Q \alpha$ for any axiom α. Compared with four-valued description logic, QC entailment holds three basic inference rules such as modus ponens: $\{C(a), C \sqsubseteq D\} \models_Q D(a)$, modus tollens: $\{\neg D(a), C \sqsubseteq D\} \models_Q \neg C(a)$ and disjunctive syllogism: $\{\neg C(a), C \sqcup D\} \models_Q D(a)$. So as a paraconsistent description

logic, QC description logic based on QC entailment has the approximate ability of classical description logic in reasoning.
- A complement of concept is defined to QC semantics in order to reverse both the information of being true and of being false. The complement of concept plays the same role as negation in classical description logics.
- A tableau algorithm for QC $\mathcal{ALCNQ}$ called QC tableau algorithm based on the complement of concept is proposed to implement paraconsistent reasoning in ontologies. QC transformation rules are developed from transformation rules in classical description logics by modifying them moderately. Furthermore, we prove that QC tableau algorithm for QC $\mathcal{ALCNQ}$ is terminable, sound and complete. Finally, we show that the complexity of QC tableau algorithm for QC $\mathcal{ALCNQ}$ is not higher than the complexity of tableau algorithms for $\mathcal{ALCNQ}$.

The paper is structured as follows. In Section 2, description logic $\mathcal{ALCNQ}$ as basic knowledge is briefly reviewed. In Section 3, we introduce QC semantics including QC weak semantics and QC strong semantics for description logic $\mathcal{ALCNQ}$ and define the QC entailment relationship. In Section 4, a terminable, sound and complete QC tableau algorithm with blocking technique to implement the querying. In Section 5, some related works on comparing the QC description logic to four-valued description logic are discussed. In Section 6, we conclude the main contributions in this paper and discuss the future work. Due to paper space limitations, proofs are absent, and detailed proofs are available in an Online Appendix.[1]

2 Preliminaries

In this section, we briefly review notation and terminology of the description logic $\mathcal{ALCNQ}$, but we basically assume that the reader is familiar with description logics. For comprehensive background reading, please refer to [17].

We assume that we are given a set of atomic concepts (or concept names), denoted by N_C, a set of roles (or role names), denoted by N_R and a set of individuals, denoted by N_I. With the symbols $\top$ and $\bot$, we furthermore denote the top concept and the bottom concept, respectively.

Complex concepts in $\mathcal{ALCNQ}$ can be formed from these inductively as follows.

- $\top$, $\bot$, and each atomic concept are concepts;
- If C, D are concepts, then $C \sqcup D$, $C \sqcap D$, and $\neg C$ are concepts;
- If C is a concept and R is a role, then $\forall R.C$, $\exists R.C$, $\geq nR.C$ and $\leq nR.C$ with n a non-negative integer are concepts.

Let C, D be concepts, a, b individuals and R a role. In description logic $\mathcal{ALCNQ}$, an ontology $\mathcal{O}$ is a pair $\langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ is called the $TBox$ (or $terminology$) of the ontology and $\mathcal{A}$ is called the $ABox$ of the ontology. $Assertions$ are of the form $C(a)$ or $R(a, b)$. $Inclusion\ axioms$ are of the form $C \sqsubseteq D$ which is called a $general\ concept\ inclusion$ (or GCI) axiom. Informally, an axiom $C(a)$ means that the individual a is an instance of concept C, and an axiom $R(a, b)$ means that individual a is related with individual b via the property R. The inclusion axiom $C \sqsubseteq D$ means that each individual of C is an individual of D.

[1] http://www.is.pku.edu.cn/~zxw/QCALCNQAppendix.pdf

Table 1. Syntax and semantics of concept constructors and axioms in $\mathcal{ALCNQ}$

Constructor Name	Syntax	Semantics
atomic concept A	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
abstract role R	R	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
individuals I	o	$o^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
inverse role	R^{-}	$(R^{-})^{I} \subseteq \Delta^{I} \times \Delta^{I}$
top concept	$\top$	Δ^{I}
bottom concept	$\bot$	$\emptyset$
conjunction	$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
disjunction	$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{I}$
exists restriction	$\exists R.C$	$\{x \mid \exists y, (x,y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$
value restriction	$\forall R.C$	$\{x \mid \forall y, (x,y) \in R^{\mathcal{I}}$ implies $y \in C^{\mathcal{I}}\}$
number atleast restriction	$\geq nR$	$\{x \mid \sharp(\{y.(x,y) \in R^{I}\}) \geq n\}$
number atmost restriction	$\leq nR$	$\{x \mid \sharp(\{y.(x,y) \in R^{I}\}) \leq n\}$
qualifying atleast restriction	$\geq nR.C$	$\{x \mid \sharp(\{y.(x,y) \in R^{I}\}$ and $y \in C^{\mathcal{I}}) \geq n\}$
qualifying atmost restriction	$\leq nR.C$	$\{x \mid \sharp(\{y.(x,y) \in R^{I}\}$ and $y \in C^{\mathcal{I}}) \leq n\}$
Axiom Name	**Syntax**	**Semantics**
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$
concept inclusion	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
individual equality	$a = b$	$a^{\mathcal{I}} = b^{\mathcal{I}}$
individual inequality	$a \neq b$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$

Remark 1. In Table 1, number restrictions can be taken as qualifying restrictions, i.e., $\leq nR \equiv \leq nR.\top$ and $\geq nR \equiv \geq nR.\top$, where $\equiv$ is the logically equivalent relationship.

The formal definition of the (model-theoretic) semantics of $\mathcal{ALCNQ}$ is given by means of interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consisting of a non-empty domain $\Delta^{\mathcal{I}}$ and a mapping $\cdot^{\mathcal{I}}$ satisfying the conditions in Table 1 where $\sharp M$ denotes the cardinality of a set M, interpreting concepts as subsets of the domain and roles as binary relations on the domain. An interpretation $\mathcal{I}$ satisfies $C \sqsubseteq D$ (written $\mathcal{I} \models C \sqsubseteq D$) iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ and it satisfies a $TBox$ $\mathcal{T}$ if it satisfies every assertion in $\mathcal{T}$. Such an interpretation is called a $model$ of $\mathcal{T}$ (written $\mathcal{I} \models \mathcal{T}$). A concept C is called $satisfiable$ w.r.t. a terminology $\mathcal{T}$ iff there is a model $\mathcal{I}$ of $\mathcal{T}$ with $C^{\mathcal{I}} \neq \emptyset$. A concept D $subsumes$ a concept C w.r.t $\mathcal{T}$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for each model $\mathcal{I}$ of $\mathcal{T}$. For an interpretation $\mathcal{I}$, an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$ is called an $instance$ of a concept C iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$. An interpretation $\mathcal{I}$ $satisfies$ $C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$, it satisfies $R(a,b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$ and it $satisfies$ an ABox $\mathcal{A}$ (written $\mathcal{I} \models \mathcal{A}$) if it satisfies every assertion in $\mathcal{A}$. An interpretation $\mathcal{I}$ $satisfies$ an $\mathcal{ALCNQ}$ ontology (i.e. is a model of the ontology) (written $\mathcal{I} \models \mathcal{O}$) iff it satisfies each axiom in both the ABox and the TBox.

An ontology is called $satisfiable$ ($unsatisfiable$) iff there exists (does not exist) such a model. An ABox is $consistent$ iff it has a model; and $inconsistent$ otherwise. Using the definition of satisfiability, an assertion α is said to be a $logical\ consequence$

of an ontology $\mathcal{O}$ (written $\mathcal{O} \models \alpha$) iff α is satisfied by every interpretation that satisfies $\mathcal{O}$. In $\mathcal{ALCNQ}$, reasoning tasks, i.e. the derivation of logical consequences, can be reduced to check satisfiability of ontologies. Tableau algorithms can implement checking satisfiability of ABox. Ian Horrocks et al [18] have developed an algorithm for combined TBox and ABox reasoning in description logic based on the following lemma.

Lemma 1. *Let C, D be concepts, $\mathcal{A}$ an ABox, $\mathcal{T}$ a TBox. We define*

$$C_{\mathcal{T}} := \sqcap_{C_i \sqsubseteq D_i \in \mathcal{T}} \neg C_i \sqcup D_i.$$

Then the following properties hold.

1. *C is satisfiable w.r.t. $\mathcal{T}$ iff $C \sqcap C_{\mathcal{T}}$ is satisfiable.*
2. *D subsumes C w.r.t. $\mathcal{T}$ iff $C \sqcap \neg D \sqcap C_{\mathcal{T}}$ is unsatisfiable.*
3. *$\mathcal{A}$ is consistent w.r.t. $\mathcal{T}$ iff $\mathcal{A} \cup \{a : C_{\mathcal{T}} \mid a$ occurs in $\mathcal{A}\}$ is consistent.*

3 Quasi-Classical Description Logic $\mathcal{ALCNQ}$

Quasi-classical description logic is a development of quasi-classical logic as defined in [15]. We assume that the language of QC description logic $\mathcal{ALCNQ}$ is $\mathcal{L}$.

3.1 Basic Definitions

Let A be an atomic concept and R be a role. A and $\neg A$ are $concept\ literals$. A concept C is in NNF, i.e., negation ($\neg$) occurs only in front of concept names. A $role\ literal$ has the form $\forall R.C$, $\exists R.C$, $\geq nR$, $\leq nR$, $\geq nR.C$ or $\leq nR.C$ with C a concept in NNF. A $literal$ is either a concept literal or a role literal, written by L. A $clause$ is the disjunction of finite literals. Let $L_1 \sqcup \cdots \sqcup L_n$ be a clause, then $Lit(L_1 \sqcup \cdots \sqcup L_n)$ is the set of literals $\{L_1, \ldots, L_n\}$ that are in the clause. A clause is $empty\ clause$, denoted by $\Diamond$, if it has no literals. We define $\sim$ be a $complementation\ operation$ such that $\sim A$ is $\neg A$ and $\sim(\neg A)$ is A. The $\sim$ operator is used to make some definitions clearer.

Definition 1. *Let $L_1 \sqcup \cdots \sqcup L_n$ be a clause that includes a literal disjunct L_i. The $focus$ of $L_1 \sqcup \cdots \sqcup L_n$ by L_i, denoted $\otimes(L_1 \sqcup \cdots \sqcup L_n, L_i)$, is defined as the clause obtained by removing L_i from $Lit(L_1 \sqcup \cdots \sqcup L_n)$. In the case of a clause with just one disjunct, we assume $\otimes(L, L) = \bot$.*

Example 1. *Given a clause $L_1 \sqcup L_2 \sqcup L_3$, $\otimes(L_1 \sqcup L_2 \sqcup L_3, L_2) = L_1 \sqcup L_3$.*

In the following, we define strong interpretation and weak interpretation over domain $\Delta^{\mathcal{I}}$ by assigning to each concept C a pair $\langle +C, -C \rangle$ of (not necessarily disjoint) subsets of $C^{\mathcal{I}}$. Intuitively, $+C$ is the set of elements known to belong to the extension of C, while $-C$ is the set of elements known to be not contained in the extension of C. $+C$ and $-C$ are not necessarily disjoint and mutual complemental with respect to the domain. The $complemental\ set$ of a set S w.r.t. an interpretation $\mathcal{I}$, denoted by $\overline{S}$, if $\overline{S} = \Delta^{\mathcal{I}} \setminus S$.

Definition 2. *Let $\mathcal{I}$ be a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ with $\Delta^{\mathcal{I}}$ as domain, where $\cdot^{\mathcal{I}}$ is a function assigning elements of $\Delta^{\mathcal{I}}$ to individuals, and subsets of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ to concepts. $\mathcal{I}$ is a $weak\ interpretation$ in QC $\mathcal{ALCNQ}$ if the conditions in Table 2 are satisfied.*

Table 2. Weak Semantics of QC $\mathcal{ALCNQ}$

Constructor Syntax	Weak Semantics
A	$A^{\mathcal{I}} = \langle +A, -A \rangle$, where $+A, -A \subseteq \Delta^{\mathcal{I}}$
R	$R^{\mathcal{I}} = \langle +R, -R \rangle$, where $+R, -R \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
o	$o^{\mathcal{I}} \in \Delta^{I}$
$\top$	$\langle \Delta^{I}, \emptyset \rangle$
$\bot$	$\langle \emptyset, \Delta^{I} \rangle$
$C_1 \sqcap C_2$	$\langle +C_1 \cap +C_2, -C_1 \cup -C_2 \rangle$,if $C_i^{\mathcal{I}} = \langle +C_i, -C_i \rangle$ for $i = 1, 2$
$C_1 \sqcup C_2$	$\langle +C_1 \cup +C_2, -C_1 \cap -C_2 \rangle$,if $C_i^{\mathcal{I}} = \langle +C_i, -C_i \rangle$ for $i = 1, 2$
$\neg C$	$(\neg C)^{\mathcal{I}} = \langle -C, +C \rangle$, if $C^{\mathcal{I}} = \langle +C, -C \rangle$
$\exists R.C$	$\langle \{x \mid \exists y, (x, y) \in +R$ and $y \in +C\}, \{x \mid \forall y, (x, y) \in +R$ implies $y \in -C\} \rangle$, if $C^{\mathcal{I}} = \langle +C, -C \rangle$ and $R^{\mathcal{I}} = \langle +R, -R \rangle$
$\forall R.C$	$\langle \{x \mid \forall y, (x, y) \in +R$ implies $y \in +C\}, \{x \mid \exists y, (x, y) \in +R$ and $y \in -C\} \rangle$, if $C^{\mathcal{I}} = \langle +C, -C \rangle$ and $R^{\mathcal{I}} = \langle +R, -R \rangle$
$\geq nR$	$\langle \{x \mid \sharp(\{y.(x, y) \in +R\}) \geq n\}, \{x \mid \sharp(\{y.(x, y) \in +R\}) < n\} \rangle$, if $R^{\mathcal{I}} = \langle +R, -R \rangle$
$\leq nR$	$\langle \{x \mid \sharp(\{y.(x, y) \in +R\}) \leq n\}, \{x \mid \sharp(\{y.(x, y) \in +R\}) > n\} \rangle$, if $R^{\mathcal{I}} = \langle +R, -R \rangle$
$\geq nR.C$	$\langle \{x \mid \sharp(\{y.(x, y) \in +R\}$ and $y \in +C) \geq n\}$, $\{x \mid \sharp(\{y.(x, y) \in +R\}$ and $y \notin -C) < n\} \rangle$ if $C^{\mathcal{I}} = \langle +C, -C \rangle$ and $R^{\mathcal{I}} = \langle +R, -R \rangle$
$\leq nR.C$	$\langle \{x \mid \sharp(\{y.(x, y) \in +R\}$ and $y \notin -C) \leq n\}$, $\{x \mid \sharp(\{y.(x, y) \in +R\}$ and $y \in +C) > n\} \rangle$ if $C^{\mathcal{I}} = \langle +C, -C \rangle$ and $R^{\mathcal{I}} = \langle +R, -R \rangle$

Definition 3. *Let $\models_w$ be a satisfiability relation called $weak$ $satisfaction$. For a weak interpretation $\mathcal{I}$, we define $\models_w$ as follows:*

- $\mathcal{I} \models_w C(a)$ *iff* $a^{\mathcal{I}} \in +C, C^{\mathcal{I}} = \langle +C, -C \rangle$*;*
- $\mathcal{I} \models_w R(a, b)$ *iff* $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in +R, R^{\mathcal{I}} = \langle +R, -R \rangle$*;*
- $\mathcal{I} \models_w C_1 \sqsubseteq C_2$ *iff* $+C_1 \subseteq +C_2$*, for* $i = 1, 2$*,* $C_i^{\mathcal{I}} = \langle +C_i, -C_i \rangle$*;*
- $\mathcal{I} \models_w a = b$ *iff* $a^{\mathcal{I}} = b^{\mathcal{I}}$*;*
- $\mathcal{I} \models_w a \neq b$ *iff* $a^{\mathcal{I}} \neq b^{\mathcal{I}}$*;*

where C, C_1, C_2 are concepts, R is a role and a is an individual.

A weak interpretation $\mathcal{I}$ $satisfies$ a terminology $\mathcal{T}$ iff $\mathcal{I} \models_w C \sqsubseteq D$ for each GCI $C \sqsubseteq D$ in $\mathcal{T}$. Such a weak interpretation is called a $QC\ weak\ model$ of $\mathcal{T}$ (written $\mathcal{I} \models_w \mathcal{T}$). A weak interpretation $\mathcal{I}$ $satisfies$ an ABox $\mathcal{A}$ iff $\mathcal{I} \models_w \alpha$ for each assertion α in $\mathcal{A}$, where $\alpha \in \{C(a), R(a, b), a = b, a \neq b\}$. Such a weak interpretation is called a $QC\ weak\ model$ of $\mathcal{A}$ (written $\mathcal{I} \models_w \mathcal{A}$).

The definition of strong interpretation is similar to weak interpretation. The main difference is that the definition for disjunction of concept in strong interpretation is more restricted.

Definition 4. *Let $\mathcal{I}$ be a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ with $\Delta^{\mathcal{I}}$ as domain, where $\cdot^{\mathcal{I}}$ is a function assigning elements of $\Delta^{\mathcal{I}}$ to individuals, and subsets of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ to concepts. $\mathcal{I}$ is a* *strong interpretation in QC $\mathcal{ALCNQ}$ if the conditions in Table 2, except conjunction concept and disjunction concept, are satisfied and $\mathcal{I}$ satisfies as follows:*

- *conjunction of concept $C_1 \sqcap C_2$: $(C_1 \sqcap C_2)$ under strong interpretation $\mathcal{I}$ is defined as $\langle +C_1 \cap +C_2, (-C_1 \cup -C_2) \cap (-C_1 \cup \overline{+C_2}) \cap (\overline{+C_1} \cup -C_2)\rangle$, if $C_i^{\mathcal{I}} = \langle +C_i, -C_i\rangle$ for $i = 1, 2$;*
- *disjunction of concept $C_1 \sqcup C_2$: $(C_1 \sqcup C_2)$ under strong interpretation $\mathcal{I}$ is defined as $\langle (+C_1 \cup +C_2) \cap (\overline{-C_1} \cup +C_2) \cap (+C_1 \cup \overline{-C_2}), -C_1 \cap -C_2\rangle$, if $C_i^{\mathcal{I}} = \langle +C_i, -C_i\rangle$ for $i = 1, 2$.*

Remark 2. In Definition 4, the disjunction of concept under strong interpretation is defined in order to hold three inference rules and the conjunction of concept under strong interpretation is defined in order to hold De Morgan law with disjunction of concept.

Definition 5. *Let $\models_s$ be a satisfiability relation called $strong\ satisfaction$. For a strong interpretation $\mathcal{I}$, we define $\models_s$ as follows:*

- *$\mathcal{I} \models_s C(a)$ iff $a^{\mathcal{I}} \in +C$ where $C^{\mathcal{I}} = \langle +C, -C\rangle$;*
- *$\mathcal{I} \models_s R(a, b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in +R$ where $R^{\mathcal{I}} = \langle +R, -R\rangle$;*
- *$\mathcal{I} \models_s C_1 \sqsubseteq C_2$ iff $\overline{-C_1} \subseteq +C_2$, $+C_1 \subseteq +C_2$ and $-C_2 \subseteq -C_1$, for $i = 1, 2$, $C_i^{\mathcal{I}} = \langle +C_i, -C_i\rangle$;*
- *$\mathcal{I} \models_s a = b$ iff $a^{\mathcal{I}} = b^{\mathcal{I}}$;*
- *$\mathcal{I} \models_s a \neq b$ iff $a^{\mathcal{I}} \neq b^{\mathcal{I}}$;*

where C, C_1, C_2 are concepts, R is a role and a is an individual.

A strong interpretation $\mathcal{I}$ $satisfies$ a terminology $\mathcal{T}$ iff $\mathcal{I} \models_s C \sqsubseteq D$ for each GCI $C \sqsubseteq D$ in $\mathcal{T}$. Such a strong interpretation is called a $QC\ strong\ model$ of $\mathcal{T}$ (written $\mathcal{I} \models_s \mathcal{T}$). A strong interpretation $\mathcal{I}$ $satisfies$ an ABox $\mathcal{A}$ iff $\mathcal{I} \models_s \alpha$ for each assertion α in $\mathcal{A}$, where $\alpha \in \{C(a), R(a, b), a = b, a \neq b\}$. Such a strong interpretation is called a $QC\ strong\ model$ of $\mathcal{A}$ (written $\mathcal{I} \models_s \mathcal{A}$).

By Definition 4 and Definition 5, we conclude several properties in the following theorems.

Theorem 1. *Let $L_1 \sqcup \cdots \sqcup L_n$ be a clause and a be an individual. $\mathcal{I} \models_s L_1 \sqcup \cdots \sqcup L_n(a)$ iff for some $L_i \in Lit(L_1 \sqcup \cdots \sqcup L_n), a^{\mathcal{I}} \in +L_i$ and $a^{\mathcal{I}} \notin -L_i$; or for all $L_i \in Lit(L_1 \sqcup \cdots \sqcup L_n), a^{\mathcal{I}} \in +L_i$ and $a^{\mathcal{I}} \in -L_i$; where $L_i^{\mathcal{I}} = \langle +L_i, -L_i\rangle$.*

Theorem 2. *Let C be a concept and R be a role. For any weak (or strong) interpretation $\mathcal{I}$ defined in Definition 2 (or Definition 4), we have*

1. *$(\neg(\leq nR.C))^{\mathcal{I}} = (> nR.C)^{\mathcal{I}}$ and $(\neg(\geq nR.C))^{\mathcal{I}} = (< nR.C)^{\mathcal{I}}$;*
2. *$(\exists R.C)^{\mathcal{I}} = (\geq 1R.C)^{\mathcal{I}}$ and $(\forall R.C)^{\mathcal{I}} = (< 1R.\neg C)^{\mathcal{I}}$.*

Theorem 3. *Let $\mathcal{I}$ and α be an interpretation and a formula in $\mathcal{ALCNQ}$ respectively.*

$$\text{If } \mathcal{I} \models_s \alpha \text{ then } \mathcal{I} \models_w \alpha.$$

Theorem 3 shows that a strong model is a weak model but not vice versa.

Example 2. *Given an ABox $\mathcal{A} = \{C(a), \neg C(a)\}$. We assume that there exists an interpretation $\mathcal{I}$ such that $a^{\mathcal{I}} \in +C$ and $a^{\mathcal{I}} \in -C$ where $C^{\mathcal{I}} = \langle +C, -C \rangle$. Clearly, $\mathcal{I} \models_w C \sqcup D(a)$ while $\mathcal{I} \not\models_s C \sqcup D(a)$.*

Definition 6. *Given a set of formula $\mathcal{K}$ and an axiom α in $\mathcal{ALCNQ}$, $\mathcal{K}$ QC $entails$ α, denoted by $\mathcal{K} \models_Q \alpha$, iff for every interpretation $\mathcal{I}$, if $\mathcal{I}$ is a strong model of every formula of $\mathcal{K}$ then $\mathcal{I}$ is a weak model of α. In this case, $\models_Q$ is a QC $entailment$ $(relation)$ between $\mathcal{K}$ and α.*

That $\models_Q$ is non-trivializable in the sense that when an ontology $\mathcal{O}$ is classically inconsistent, it is not the case that any axiom is entailed by $\mathcal{O}$ in QC $\mathcal{ALCNQ}$.

Example 3. *Given an ABox $\mathcal{A} = \{B(a), \neg B(a)\}$ and an atomic concept A in QC $\mathcal{ALCNQ}$. So $\{B(a), \neg B(a)\}$ is classically inconsistent. However it is not the case that $\mathcal{A} \models_Q C(a)$ holds, since there exists an interpretation $\mathcal{I}$ such that $a^{\mathcal{I}} \in +B$ and $a^{\mathcal{I}} \in -B$ where $B^{\mathcal{I}} = \langle +B, -B \rangle$. So $\mathcal{I} \models_s B \sqcap \neg B(a)$, but $\mathcal{I} \not\models_w A(a)$ since $A(a)$ does not occur in $\mathcal{A}$.*

Example 4. *Let an ABox $\mathcal{A} = \{B \sqcup C(a), \neg B(a)\}$. For all interpretations $\mathcal{I}$ such that ($a^{\mathcal{I}} \in +B$ or $a^{\mathcal{I}} \in +C$) and $a^{\mathcal{I}} \in -B$, if $\mathcal{I} \models_s B \sqcup C(a)$ and $\mathcal{I} \models_s \neg B(a)$ then $\mathcal{I} \models_w C(a)$. Hence, $\mathcal{A} \models_Q B \sqcup C(a)$, $\mathcal{A} \models_Q \neg B(a)$, and $\mathcal{A} \models_Q C(a)$.*

Example 5. *Suppose an ABox $\mathcal{A}$ is empty. Now consider the classical $\top = A \sqcup \neg A$. Here $\mathcal{A} \models_Q A \sqcup \neg A(a)$ does not hold. Since $\mathcal{A}$ strongly satisfies every formula in $\mathcal{A}$, but $\mathcal{A}$ does not weakly satisfy $A \sqcup \neg A(a)$.*

Example 5 shows that QC description logic is weaker than description logic in ability of reasoning.

Definition 7. *A $complement$ of a concept C is defined by $\overline{C}^{\mathcal{I}} = \langle \overline{+C}, \overline{-C} \rangle$ if $C^{\mathcal{I}} = \langle +C, -C \rangle$ where $\mathcal{I}$ is a (strong or weak) interpretation of C, denoted by $\overline{C}$. We call $\overline{C}$ is the $complement$ of C.*

Remark 3. The intuition behind complement of concepts is to reverse both the information of being true and of being false. And any $\mathcal{ALCNQ}$ ABox doesn't contain complement concepts.

In QC $\mathcal{ALCNQ}$, the set of signed formulae of $\mathcal{L}$ is denoted $\mathcal{L}^*$ and is defined as $\mathcal{L} \cup \{\overline{C} | C \in \mathcal{L}\}$. In the following, we discuss the QC tableau algorithm based on the language $\mathcal{L}^*$.

Theorem 4. *For concept name A, concepts C, D in N_C, individuals a, b in N_I and a role R in N_R in QC $\mathcal{ALCNQ}$.*

(1) $\mathcal{I} \models_x \overline{A}(a)$ *iff* $\mathcal{I} \not\models_x A(a)$
(2) $\mathcal{I} \models_x \overline{\neg A}(a)$ *iff* $\mathcal{I} \models_x \neg \overline{A}(a)$
(3) $\mathcal{I} \models_x \overline{\overline{A}}(a)$ *iff* $\mathcal{I} \models_x A(a)$
(4) $\mathcal{I} \models_w \overline{C} \sqcup \overline{D}(a)$ *iff* $\mathcal{I} \models_w \overline{C \sqcap D}(a)$
(5) $\mathcal{I} \models_w \overline{C} \sqcap \overline{D}(a)$ *iff* $\mathcal{I} \models_w \overline{C \sqcup D}(a)$
(6) $\mathcal{I} \models_x \overline{\top}(a)$ *iff* $\mathcal{I} \models_x \bot(a)$
(7) $\mathcal{I} \models_x \overline{(\forall R.C)}(a)$ *iff* $\mathcal{I} \models_x \exists R.\overline{C}(a)$
(8) $\mathcal{I} \models_x \overline{(\exists R.C)}(a)$ *iff* $\mathcal{I} \models_x \forall R.\overline{C}(a)$
(9) $\mathcal{I} \models_x \overline{(\geq nR)}(a)$ *iff* $\mathcal{I} \models_x \leq n-1R(a)$
(10) $\mathcal{I} \models_x \overline{(\leq nR)}(a)$ *iff* $\mathcal{I} \models_x \geq n+1R(a)$
(11) $\mathcal{I} \models_x \overline{(\geq nR.C)}(a)$ *iff* $\mathcal{I} \models_x \leq n-1R.C(a)$
(12) $\mathcal{I} \models_x \overline{(\leq nR.C)}(a)$ *iff* $\mathcal{I} \models_x \geq n+1R.C(a)$

Here $\models_x$ is a place-holder for both $\models_w$ and $\models_s$.

Theorem 5. *Let $\mathcal{I}$ be an interpretation and let C, D be concepts.*

1. $\mathcal{I} \models_s C \sqsubseteq D$ *iff for any individual* $a \in N_I$, $\mathcal{I} \models_s \neg C \sqcup D(a)$.
2. $\mathcal{I} \models_w C \sqsubseteq D$ *iff for any individual* $a \in N_I$, $\mathcal{I} \models_w \overline{C} \sqcup D(a)$.

In QC $\mathcal{ALCNQ}$, a $QC\ ABox$ ($QC\ TBox$) is a set whose axioms (concept inclusions) are formulae in $\mathcal{L}^*$. A concept C is $QC\ satisfiable$ w.r.t. a QC ABox $\mathcal{A}$ if there exists an individual a such that $\mathcal{A} \models_Q C(a)$, and $QC\ unsatisfiable$ w.r.t. $\mathcal{A}$ otherwise. A concept C is $QC\ satisfiable$ w.r.t. a QC ABox $\mathcal{A}$ and a QC TBox $\mathcal{T}$ if there exists an individual a such that $\mathcal{T} \cup \mathcal{A} \models_Q C(a)$, and $QC\ unsatisfiable$ w.r.t. $\mathcal{T}$ and $\mathcal{A}$ otherwise. A QC ABox $\mathcal{A}$ is QC $consistent$ if each concept occurring in $\mathcal{A}$ is QC satisfiable w.r.t. $\mathcal{A}$, and $QC\ inconsistent$ otherwise. A QC ABox $\mathcal{A}$ is $QC\ consistent$ w.r.t. a QC TBox $\mathcal{T}$ if each concept occurring in $\mathcal{A}$ is QC satisfiable w.r.t. $\mathcal{T}$ and $\mathcal{A}$, and $QC\ inconsistent$ w.r.t. $\mathcal{T}$ otherwise. A set of QC ABoxes $\mathcal{S}$ is $QC\ consistent$ iff there is a QC satisfiable QC ABox in $\mathcal{S}$, $QC\ inconsistent$ otherwise. A set of QC ABoxes $\mathcal{S}$ is $QC\ satisfiable$ iff there is a QC satisfiable QC ABox in $\mathcal{S}$, $QC\ unsatisfiable$ otherwise.

A concept C is in $QC\ NNF$, i.e., iff complement only occurs over concept names. By Theorem 4, for any concept $C \in \mathcal{L}^*$, $\overline{C}$ can be transformed into its QC NNF. For a concept, we define $clos(C)$ as the smallest set that contains C and is closed under sub-concepts, its NNF and QC NNF of its complement. We denote $clos(\mathcal{A}) = \bigcup_{C \in \mathcal{A}} clos(C)$. Clearly, the size of $clos(\mathcal{A})$ is polynomial in the size of $\mathcal{A}$.

Theorem 6. *Let $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ be an $\mathcal{ALCNQ}$ ontology, a be a individual and ι be a new individual not occurring in $\mathcal{O}$. Then the following hold.*

1. $\mathcal{A} \models_Q C(a)$ *iff* $\mathcal{A} \cup \{\overline{C}(a)\}$ *is QC unsatisfiable.*
2. $\mathcal{A} \models_Q C \sqsubseteq D$ *iff* $\mathcal{A} \cup \{C \sqcap \overline{D}(\iota)\}$ *is QC unsatisfiable.*

4 QC Tableau Algorithm for $\mathcal{ALCNQ}$

In this section, we develop a tableau algorithm called by $QC\ tableau\ algorithm$ for reasoning with inconsistency in QC $\mathcal{ALCNQ}$. $R_{\mathcal{A}}$ denotes the set of roles occurring in $\mathcal{A}$, and $U_{\mathcal{A}}$ denotes the set of individuals occurring in $\mathcal{A}$.

4.1 QC Tableau

Definition 8. *Given a QC ABox $\mathcal{A}$, $T = (S, \mathcal{L}, \mathcal{E}, \mathcal{J})$ is a QC tableau for $\mathcal{A}$ iff*

- S *is a non-empty set;*
- $\mathcal{L} : S \to 2^{clos(\mathcal{A})}$ *maps each element in S to a set of concepts;*
- $\mathcal{E} : R_{\mathcal{A}} \to 2^{S \times S}$ *maps each role to a set of pairs of elements in S;*
- $\mathcal{J} : U_{\mathcal{A}} \to S$ *maps individuals occurring in $\mathcal{A}$ to elements in S.*

Furthermore, for all $s, t \in S, C, C_1, C_2 \in clos(\mathcal{A})$ and T satisfies:

(P1) if $C \in \mathcal{L}(s)$*, then* $\overline{C} \notin \mathcal{L}(s)$*,*
(P2) if $C_1 \sqcap C_2 \in \mathcal{L}(s)$*, then* $C_1 \in \mathcal{L}(s)$ *and* $C_2 \in \mathcal{L}(s)$*,*

(P3) if $C_1 \sqcup C_2 \in \mathcal{L}(s)$*, then*
(a) if $\sim C_i \in \mathcal{L}(s)$ *for some (*$i \in \{1, 2\}$*), then* $\otimes(C_1 \sqcup C_2, C_i) \in \mathcal{L}(s)$*,*
(b) else $C_1 \in \mathcal{L}(s)$ *or* $C_2 \in \mathcal{L}(s)$*,*
(P4) if $\forall R.C \in \mathcal{L}(s)$ *and* $\langle s, t\rangle \in \mathcal{E}(R)$*, then* $C \in \mathcal{L}(t)$*,*
(P5) if $\exists R.C \in \mathcal{L}(s)$*, then there is some* $t \in S$ *such that* $\langle s, t\rangle \in \mathcal{E}(R)$ *and* $C \in \mathcal{L}(t)$*,*
(P6) if $\leq nR.C \in \mathcal{L}(s)$*, then* $\sharp R^T(s, C) \leq n$*,*
(P7) if $\geq nR.C \in \mathcal{L}(s)$*, then* $\sharp R^T(s, C) \geq n$*,*
(P8) if $(\bowtie nR.C) \in \mathcal{L}(s)$ *and* $\langle s, t\rangle \in \mathcal{E}(R)$ *then* $C \in \mathcal{L}(t)$ *or* $\overline{C} \in \mathcal{L}(t)$*,*
(P9) if $a : C \in \mathcal{A}$*, then* $C \in \mathcal{L}(\mathcal{J}(a))$*,*
(P10) if $(a, b) : R \in \mathcal{A}$*, then* $\langle \mathcal{J}(a), \mathcal{J}(b)\rangle \in \mathcal{E}(R)$*,*
(P11) if $a \neq b \in \mathcal{A}$*, then* $\mathcal{J}(a) \neq \mathcal{J}(b)$*,*

where $\bowtie$ *is a place-holder for both* $\leq$ *and* $\geq$*, and* $R^T(s, C) = \{t \in S \mid \langle s, t\rangle \in \mathcal{E}(R)$ *and* $C \in \mathcal{L}(t)\}$*.*

Theorem 7. *A QC ABox* $\mathcal{A}$ *is QC satisfiable iff there is a QC tableau for* $\mathcal{A}$*.*

4.2 QC Tableau Algorithm

In this subsection, we develop a QC tableau algorithm based on QC transformation rules which are obtained by modifying the expansion rules in [18] in order to obtain a QC tableau for a QC ABox $\mathcal{A}$. In the following, we borrow the reasoning technique from Horrocks [18,19] to implement paraconsistent reasoning under QC semantics.

In QC $\mathcal{ALCNQ}$, a completion forest $\mathcal{F}$ for a QC ABox $\mathcal{A}$ is a collection of trees whose distinguished root nodes are possibly connected by edges in an arbitrary way. Moreover, each node x is labeled with a set $\mathcal{L}(x) \subseteq clos(\mathcal{A})$ and each edge $\langle x, y\rangle$ is labeled with a set $\mathcal{L}(\langle x, y\rangle) \in R$ of (possibly inverse) roles occurring in $\mathcal{A}$. Finally, completion forests come with an explicit inequality relation $\neq$ on nodes and an explicit equality relation $=$ which are implicitly assumed to be symmetric.

If nodes x and y are *connected* by an edge $\langle x, y\rangle$ with $R \in \mathcal{L}(\langle x, y\rangle)$, then y is called an R-*successor* of x and x is called an R-*predecessor* of y. If y is an R-successor of x, then y is called an R-*neighbor* of x. A node y is a *successor* (resp. *predecessor* or *neighbor*) of y if it is an R-successor (resp. R-predecessor or R-neighbor) of y for some role R. Finally, *ancestor* is the transitive closure of predecessor.

For a role R, a concept C and a node x in $\mathcal{F}$ we define $R^{\mathcal{F}}(x, C)$ by $R^{\mathcal{F}}(x, C) := \{y \mid y \text{ is } R\text{-neighbor of } x \text{ and } C \in \mathcal{L}(y)\}$. A node is *blocked* iff it is not a root node and it is either directly or indirectly blocked. A node x is *directly blocked* iff none of its ancestors are blocked, and it has ancestors x', y and y' such that

- y is not a root node;
- x is a successor of x' and y is a successor of y';
- $\mathcal{L}(x) = \mathcal{L}(y)$ and $\mathcal{L}(x') = \mathcal{L}(y')$;
- $\mathcal{L}(\langle x', x\rangle) = \mathcal{L}(\langle y', y\rangle)$.

In QC $\mathcal{ALCNQ}$, given a QC ABox $\mathcal{A}$ and a role R, the algorithm initializes a completion forest $\mathcal{F}_{\mathcal{A}}$ consisting only of root nodes. More precisely, $\mathcal{F}_{\mathcal{A}}$ contains a root node x_0^i for each individual $a^i \in U_{\mathcal{A}}$, and an edge $\langle x_0^i, x_0^j\rangle$ if $\mathcal{A}$ contains an assertion $(a_i, a_j) : R$ for some R. The labels of these nodes and edges and the relations $\neq$ and $=$ are initialized as follows:

(1) $\mathcal{L}(x_i^0) := \{C \mid a_i : C \in \mathcal{A}\}$, and
(2) $\mathcal{L}(\langle x_0^i, x_0^j \rangle) := \{R \mid (a_i, a_j) : R \in \mathcal{A}\}, x_0^i \neq x_0^j$ iff $a_i \neq a_j \in \mathcal{A}$, and the $=$ -relation is initialized to be empty.

$\mathcal{F}_\mathcal{A}$ is then expanded by repeatedly applying the rules from Table 3.

Definition 9. *Let $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ be an ontology, C_1, C_2, C concepts, R a role and x, y, z individuals in QC $\mathcal{ALCNQ}$. We define QC $transformation\ rules$ in Table 3.*

For a node x, $\mathcal{L}(x)$ is said to contain a $clash$ if $\{A, \overline{A}\} \in \mathcal{L}(x)$ for some concept name $A \in N_C$, or if there is some concept $\leq nR.C \in \mathcal{L}(x)$ and x has $n+1$ R-neighbors $y_0, \ldots, y_n$ with $C \in \mathcal{L}(y_i)$ and $y_i \neq y_j$ for all $0 \leq i < j \leq n$. A completion forest is $clash\text{-}free$ if none of its nodes contains a clash, and it is complete if no QC transformation rule from Table 3 can be applied to it. For a QC ABox $\mathcal{A}$, the algorithm starts with the completion forest $\mathcal{F}_\mathcal{A}$. It applies the QC transformation rules in Table 3, stopping when a clash occurs, and answers " $\mathcal{A}$ is QC satisfiable " iff the QC transformation rules can be applied in such a way that they yield a complete and clash-free completion forest; and answering " $\mathcal{A}$ is QC unsatisfiable " otherwise.

Theorem 8. *Let $\mathcal{A}$ be a QC ABox in QC $\mathcal{ALCNQ}$.*

1. *The QC tableau algorithm terminates when started for $\mathcal{A}$.*
2. *If the QC transformation rules can be applied to $\mathcal{A}$ such that they yield a complete and clash-free completion forest, then $\mathcal{A}$ has a QC tableau.*
3. *If $\mathcal{A}$ has a QC tableau, then the QC transformation rules can be applied to $\mathcal{A}$ such that they yield a complete and clash-free completion forest.*
4. *The QC tableau algorithm is a decision procedure for the consistency of QC ABoxes and the satisfiability and subsumption of concepts with respect to terminologies.*

Theorem 9. *Satisfiability of QC ABoxes is PSPACE-complete in $\mathcal{ALCNQ}$.*

5 Related Work

In this section, we compare QC description logic with four-valued description logic [7,9,20] which is a paraconsistent logic by integrating description logic with Belanp's four-valued logic [12]. The similarities and differences between QC logic and four-valued logic still exist between QC DLs and four-valued DLs.

In QC DLs, the weak semantics is analogous to the four-valued semantics in four-valued DLs. In fact, inconsistent or incomplete can be tolerated under two semantics and the contradiction can be taken as a part of ontologies. Below we give a presentation of the definition 4-model of four-valued DL which was proposed in [7,9,20].

Definition 10 (Ma [9]). *Let C, D be concepts and a be an individual in four-valued DLs. An interpretation $\mathcal{I}$ is a 4-model of an axiom $C(a)$ (written by $\mathcal{I} \models_4 C(a)$) iff $a^\mathcal{I} \in proj^+(C^\mathcal{I})$; $\mathcal{I}$ is a 4-model of a material inclusion $C \mapsto D$ (written by $\mathcal{I} \models_4 C \mapsto D$) iff $\overline{proj^-(C^\mathcal{I})} \subseteq proj^+(D^\mathcal{I})$; $\mathcal{I}$ is a 4-model of an internal inclusion $C \sqsubset D$ (written by $\mathcal{I} \models_4 C \sqsubset D$) iff $proj^+(C^\mathcal{I}) \subseteq proj^+(D^\mathcal{I})$ and $\mathcal{I}$ is a 4-model of a strong inclusion $C \rightarrow D$ (written by $\mathcal{I} \models_4 C \rightarrow D$) iff $proj^+(C^\mathcal{I}) \subseteq proj^+(D^\mathcal{I})$ and $proj^-(D^\mathcal{I}) \subseteq proj^-(C^\mathcal{I})$.*

Table 3. QC Transformation Rules in QC $\mathcal{ALCNQ}$

1. The $\rightarrow_{\sqcap}$-rule Condition: $C_1 \sqcap C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\{C_1, C_2\} \not\subseteq \mathcal{L}(x)$. Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{C_1(x), C_2(x)\}$.
2. The $\rightarrow_{\sqcup}$-rule Condition: $C_1 \sqcup C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\{C_1, C_2, \sim C_1, \sim C_2\} \cap \mathcal{L}(x) = \emptyset$. Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{E\}$ for some $E \in \{C_1, C_2\}$.
3.The $\rightarrow_{QC}$-rule Condition: $C_1 \sqcup C_2 \in \mathcal{L}(x)$, x is not indirectly blocked, and $\sim C_i \in \mathcal{L}(x)$(for some $i \in \{1, 2\}$). Action: $\mathcal{L}(x) := \mathcal{L}(x) \cup \{\otimes(C_1 \sqcup C_2, C_i)\}$.
4. The $\rightarrow_{\exists}$-rule Condition: $\exists R.C \in \mathcal{L}(x)$, x is not blocked, and x has no R-neighbor y with $C \in \mathcal{L}(y)$. Action: create a new node y with $\mathcal{L}(\langle x, y\rangle) := \{R\}$ and $\mathcal{L}(y) := \{C\}$.
5. The $\rightarrow_{\forall}$-rule Condition: (1) $\forall R.C \in \mathcal{L}(x)$, x is not indirectly blocked, and (2) there is an R-neighbor y of x with $C \in \mathcal{L}(y)$. Action: $\mathcal{L}(y) := \mathcal{L}(y) \cup \{C\}$.
6. The choose-rule Condition: $(\bowtie nR.C) \in \mathcal{L}(x)$, x is not indirectly blocked, and there is an R-neighbor y of x with $\{C, \overline{C}\} \cap \mathcal{L}(y) = \emptyset$. Action: $\mathcal{L}(y) := \mathcal{L}(y) \cup \{E\}$ for some $E \in \{C, \overline{C}\}$.
7. The $\rightarrow_{\geq}$-rule Condition: (1)$\geq nR.C \in \mathcal{L}(x)$, x is not blocked, and (2) there are no n R-neighbors $y_1, \ldots, y_n$ such that $C \in \mathcal{L}(y_i)$ and $y_i \neq y_j$ for $1 \leq i < j \leq n$. Action: create n new nodes $y_1, \ldots, y_n$ with $\mathcal{L}(\langle x, y_i\rangle) := \{R\}$, $\mathcal{L}(y_i) := \{C\}$, and $y_i \neq y_j$.
8. The $\rightarrow_{\leq}$-rule Condition: (1) $\leq nR.C \in \mathcal{L}(x)$, x is not indirectly blocked, and $\sharp R^{\mathcal{F}}(x, C) > n$, (2) there are R-neighbors y, z of x with not $y \neq z$, y is neither a root node nor an ancestor of z, and $C \in \mathcal{L}(y) \cap \mathcal{L}(z)$. Action: (1)$\mathcal{L}(z) := \mathcal{L}(z) \cup \mathcal{L}(y)$; and (2)if z is an ancestor of x then $\mathcal{L}(\langle z, x\rangle) := \mathcal{L}(\langle z, x\rangle)$ else $\mathcal{L}(\langle x, z\rangle) := \mathcal{L}(\langle x, z\rangle) \cup \mathcal{L}(\langle x, y\rangle)$ (3) $\mathcal{L}(\langle x, y\rangle) := \emptyset$; (4) set $u \neq z$ for all u with $u \neq y$.
9. The $\rightarrow_{\leq_r}$-rule Condition: (1)$\leq nR.C \in \mathcal{L}(x)$, and $\sharp R^{\mathcal{F}}(x, C) > n$ and (2) there are two R-neighbors y, z of x which are both root nodes, $C \in \mathcal{L}(y) \cap \mathcal{L}(z)$, and not $y \neq z$. Action: (1) $\mathcal{L}(z) := \mathcal{L}(z) \cup \mathcal{L}(y)$; (2) for all edges $\langle y, w\rangle$: (a) if the edge $\langle z, w\rangle$ does not exist, create it with $\mathcal{L}(\langle z, w\rangle) := \emptyset$ and (b) $\mathcal{L}(\langle z, w\rangle) := \mathcal{L}(\langle z, w\rangle) \cup \mathcal{L}(\langle y, w\rangle)$; (3) for all edges $\langle w, y\rangle$: (a) if the edge $\langle w, z\rangle$ does not exist, create it with $\mathcal{L}(\langle w, z\rangle) := \emptyset$ and (b) $\mathcal{L}(\langle w, z\rangle) := \mathcal{L}(\langle w, z\rangle) \cup \mathcal{L}(\langle w, y\rangle)$; (4) set $\mathcal{L}(y) := \emptyset$ and remove all edges to/from y; (5) set $u \neq z$ for all u with $u \neq y$ and set $y \neq z$.

Definition 11 (Ma [9]). *Let $\mathcal{O}$ be an ontology and α be an axiom in four-valued DL. $\mathcal{O}$ entails α under four semantics, denoted by $\mathcal{O} \models_4 \alpha$, iff for any model $\mathcal{I}$ of $\mathcal{O}$,[2] $\mathcal{I}$ is a 4-model of α.*

The follow property shows that there exists a close relationship between weak models and 4-models in description logic.

Theorem 10. *Let C, D be concepts, a be an individual and $\mathcal{I}$ be an interpretation in description logic.*

$$1. \mathcal{I} \models_w C(a) \text{ iff } \mathcal{I} \models_4 C(a); \quad 2. \mathcal{I} \models_w C \sqsubseteq D \text{ iff } \mathcal{I} \models_4 C \sqsubset D.$$

The follow property shows that strong models is stronger than 4-models in description logic.

Theorem 11. *Let C, D be concepts, a be an individual and $\mathcal{I}$ be an interpretation in description logic.*

1. *$\mathcal{I} \models_4 C(a)$ if $\mathcal{I} \models_s C(a)$;*
2. *$\mathcal{I} \models_4 C \propto D$, where $\propto \in \{\mapsto, \sqsubset, \rightarrow\}$ if $\mathcal{I} \models_s C \sqsubseteq D$.*

Theorem 12. *Let $\mathcal{O}$ be an ontology and α be a query in $\mathcal{ALC}$. If $\mathcal{O} \models_4 \alpha$ then $\mathcal{O} \models_Q \alpha$.*

Theorem 12 shows that QC entailment is stronger than four-valued entailment because QC entailment holds modus ponens, modus tollens, hypothetical syllogism and disjunctive syllogism which four-valued entailment does hold.

6 Conclusions

In this paper, we present QC OWL to handle inconsistency with holding three inference rules and intuitive equivalences. For the aim of paraconsistent reasoning, QC weak semantics is similar to four-valued semantics for OWL. In order to hold three inference rules, QC strong semantics is introduced by restricting disjunction of concepts in description logic. Compared with four-valued OWL, we redefine concept inclusion (or subsumption) in QC OWL to follow intuitive equivalences. For this purpose, concept inclusion under QC weak semantics is defined by internal inclusion of four-valued OWL and concept inclusion under QC strong semantics is defined by hybrid of three inclusions (material inclusion, internal inclusion and strong inclusion) of four-valued OWL. In order to find suitable for implementation of paraconsistent reasoning in QC OWL, we propose the QC tableau algorithm with technique of blocking for checking satisfiability of QC $\mathcal{ALCNQ}$ ABox. Technique of blocking is introduced to hold the terminability of QC tableau algorithm. Moreover, we conclude the terminability, soundness and completeness of the algorithm. The complexity of QC tableau algorithm is not more than the complexity of classical tableau algorithm for an $\mathcal{ALCNQ}$ ABox. Complement of

[2] An interpretation is a 4-models of an ontology iff it satisfies each assertion and each inclusion axiom in the ontology [9].

concept is introduced to QC semantics in order to build reasoning system of QC OWL by not rebuilding whole new reasoning system but borrowing from classical reasoning systems of OWL. It is quite easy to show that the QC tableau algorithm, which is presented in this paper, is obtained by modifying classical reasoning systems of OWL [18]. In the future, we will further study the similar paraconsistent approach for more complex DLs such as OWL Lite and OWL DL.

References

1. Bell, D.A., Qi, G., Liu, W.: Approaches to inconsistency handling in description-logic based ontologies. In: Meersman, R., Tari, Z., Herrero, P. (eds.) OTM-WS 2007, Part II. LNCS, vol. 4806, pp. 1303–1311. Springer, Heidelberg (2007)
2. Besnard, P., del Cerro, L.F., Gabbay, D.M., Hunter, A.: Logical handling of inconsistent and default information. In: Uncertainty Management in Information Systems, pp. 325–342 (1996)
3. Huang, Z., van Harmelen, F., ten Teije, A.: Reasoning with inconsistent ontologies. In: Proceedings of the Nineteenth International Joint Conference on Artificial Intelligence, IJCAI 2005, Edinburgh, Scotland, UK, July 30-August 5, 2005, pp. 454–459. Professional Book Center (2005)
4. Patel-Schneider, P.F., Hayes, P., Horrocks, I.: Owl web ontology language semantics and abstract syntax section 5. rdf-compatible model-theoretic semantics. Technical report, W3C (December 2004)
5. Schlobach, S., Cornet, R.: Non-standard reasoning services for the debugging of description logic terminologies. In: Proceedings of the Eighteenth International Joint Conference on Artificial Intelligence, IJCAI 2003, Acapulco, Mexico, pp. 355–362. Morgan Kaufmann, San Francisco (2003)
6. Haase, P., van Harmelen, F., Huang, Z., Stuckenschmidt, H., Sure, Y.: A framework for handling inconsistency in changing ontologies. In: Gil, Y., Motta, E., Benjamins, V.R., Musen, M.A. (eds.) ISWC 2005. LNCS, vol. 3729, pp. 353–367. Springer, Heidelberg (2005)
7. Straccia, U.: A sequent calculus for reasoning in four-valued description logics. In: Galmiche, D. (ed.) TABLEAUX 1997. LNCS, vol. 1227, pp. 343–357. Springer, Heidelberg (1997)
8. Ma, Y., Hitzler, P., Lin, Z.: Algorithms for paraconsistent reasoning with owl. In: Franconi, E., Kifer, M., May, W. (eds.) ESWC 2007. LNCS, vol. 4519, pp. 399–413. Springer, Heidelberg (2007)
9. Ma, Y., Hitzler, P., Lin, Z.: Paraconsistent resolution for four-valued description logics. In: Proceedings of the 2007 International Workshop on Description Logics (DL 2007), Brixen-Bressanone, near Bozen-Bolzano, Italy, June 8-10, 2007, vol. 250. CEUR-WS.org (2007)
10. Ma, Y., Hitzler, P., Lin, Z.: Paraconsistent reasoning for expressive and tractable description logics. In: Proceedings of the 21st International Workshop on Description Logics (DL 2008), Dresden, Germany, May 13-16, 2008, CEUR-WS.org (2008)
11. Qi, G., Pan, J.Z., Ji, Q.: Extending description logics with uncertainty reasoning in possibilistic logic. In: Mellouli, K. (ed.) ECSQARU 2007. LNCS, vol. 4724, pp. 828–839. Springer, Heidelberg (2007)
12. Belnap, N.D.: A useful four-valued logic. Modern uses of multiple-valued logics, 7–73 (1977)
13. Ma, Y., Qi, G., Hitzler, P., Lin, Z.: Measuring inconsistency for description logics based on paraconsistent semantics. In: Proceedings of the 2007 International Workshop on Description Logics (DL 2007), Brixen-Bressanone, near Bozen-Bolzano, Italy, June 8-10, 2007. CEUR Workshop Proceedings, vol. 250, CEUR-WS.org (2007)

14. Zhang, X., Lin, Z.: Paraconsistent reasoning with quasi-classical semantic in alc. In: Calvanese, D., Lausen, G. (eds.) RR 2008. LNCS, vol. 5341, pp. 222–229. Springer, Heidelberg (2008)
15. Besnard, P., Hunter, A.: Quasi-classical logic: Non-trivializable classical reasoning from incositent information. In: Froidevaux, C., Kohlas, J. (eds.) ECSQARU 1995. LNCS, vol. 946, pp. 44–51. Springer, Heidelberg (1995)
16. Hunter, A.: Reasoning with contradictory information using quasi-classical logic. J. Log. Comput. 10(5), 677–703 (2000)
17. Baader, F., Calvanese, D., McGuinness, D., Nardi, D., Patel-Schneider, P. (eds.): The Description Logic Handbook: Theory, Implementation, and Applications. Cambridge University Press, Cambridge (2003)
18. Horrocks, I., Sattler, U., Tobies, S.: Reasoning with individuals for the description logic shiq. CoRR cs.LO/0005017 (2000)
19. Horrocks, I., Sattler, U.: A tableaux decision procedure for shoiq. In: Kaelbling, L.P., Saffiotti, A. (eds.) IJCAI, pp. 448–453. Professional Book Center (2005)
20. Patel-Schneider, P.F.: A four-valued semantics for terminological logics. Artificial Intelligence 38, 319–351 (1989)