Computing Inconsistency Measurements under Multi-Valued Semantics by Partial Max-SAT Solvers

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February 8, 2011

Outline

- Motivation
- Inconsistency Measures under Multi-Valued Semantics
- Relationship among Different Measurements
- Encoding Algorithms
- Conclusion and Future Work

Motivation

- Consistent KBs serve as useful knowledge resources v.s. inconsistent KBs imply any conclusion (meaningless!)
- For handling inconsistent KBs:
  - paraconsistent reasoning (1960s)
  - knowledge diagnose and repair (1980s)
  - Which approach should we take?

  $\leadsto$ inconsistency measurement: a guidance to choose different approaches (2000s)

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  - paraconsistent reasoning (1960s)
  - knowledge diagnose and repair (1980s)
  - Which approach should we take?
  - $\rightsquigarrow$ inconsistency measurement: a guidance to choose different approaches (2000s)
- Problem
  - Relationship among different measurements
  - Efficient algorithms

Definitions

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- Multi-Valued Semantics
  - 4-valued, 3-valued, LP_m, Quasi-Classical, ...
  - $I : Var(K) \rightarrow \{t, f, Both, None\}$

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- ID of K respect to I under i-semantics ($i = 3, 4, LP_m, Q$)

$$ID_i(K, I) = \frac{|\{p \mid p^I = B, p \in Var(K)\}|}{|Var(K)|}, \text{ if } I \models_i K$$

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- ID of K under under i-semantics ($i = 3, 4, LP_m, Q$)

$$ID_i(K) = \min_{I \models K} ID_i(K, I)$$

Inconsistency Degree under 4-valued Semantics

Truth values: $\{t, f, B, N\}$

4-model I:

$K \rightarrow \{t, B\}$

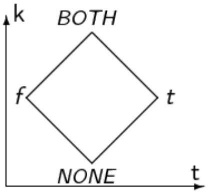

Figure: FOUR

➤ $ID_4(K, I) = \frac{|\{p|p^I = B, p \in Var(K)\}|}{|Var(K)|}$

$ID_4(K) = min_{I \models_4 K} ID_4(K),$

$\rightsquigarrow$ $K = \{p, \neg q, \neg p \vee q, r \vee s\}$

$\rightsquigarrow$ $I_1 : p^{I_1} = B, q^{I_1} = f, r^{I_1} = t, s^{I_1} = t,$

$I_2 : p^{I_2} = B, q^{I_2} = B, r^{I_2} = t, s^{I_2} = t$

$I_3 : p^{I_3} = B, q^{I_3} = B, r^{I_3} = t, s^{I_3} = N$

$\rightsquigarrow$ $ID_4(K, I_1) = \frac{1}{4}$, $ID_4(K, I_2) = \frac{2}{4}$

$ID_4(K, I_3) = \frac{2}{4}$

$ID_4(K) = \frac{1}{4}$

Inconsistency Degree under 3-valued Semantics

Truth values: $\{t, f, B\}$

3-model I:

$K \to \{t, B\}$

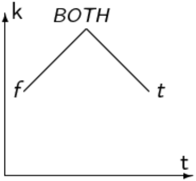

Figure: Three

- $ID_3(K, I) = \frac{|\{p \mid p^I = B, p \in Var(K)\}|}{|Var(K)|}$

  $ID_3(K) = min_{I \models_3 K} ID_3(K)$,

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$ID_3(K) = \frac{1}{4}$

Inconsistency Degree under LPm Semantics

LP_m interpretation:

- 3-valued interpretation
- only "most classical" ones are considered

- $ID_{LP_m}(K, I) = \frac{|\{p|p^I = B, p \in Var(K)\}|}{|Var(K)|}$
  $ID_{LP_m}(K) = min_{I \models_{LP_m} K} ID_{LP_m}(K)$,

$\leadsto$ $K = \{p, \neg q, \neg p \vee q, r \vee s\}$

$\leadsto$ $I_1 : p^{I_1} = B, q^{I_1} = f, r^{I_1} = t, s^{I_1} = t$,
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$ID_{LP_m}(K) = \frac{1}{4}$

Inconsistency Degree under Quasi-Classical Semantics

Quasi-Classical (Q) interpretation:

- 4-valued interpretation
- Resolution laws are satisfied

  $I \models_Q \alpha \vee \beta,$

  $I \models_Q \neg\beta \vee \gamma$

  $\Rightarrow I \models_Q \alpha \vee \gamma$

- $ID_Q(K, I) = \frac{|\{p|p^I = B, p \in Var(K)\}|}{|Var(K)|}$

  $ID_Q(K) = min_{I \models_Q K} ID_Q(K),$

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Theorem

Given a knowledge base K, then
$ID_3(K) = ID_4(K) = ID_{LP_m}(K) \leq ID_Q(K)$.

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Proof Hints:

- $ID_3(K) \geq ID_4(K)$: Trivial since $I \models_3 K \Rightarrow I \models_4 K$

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- $ID_3(K) \leq ID_4(K)$: Every 4-model can be modified to a 3-model by changing N to t while preserving the inconsistency degree.

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- $ID_3(K) \geq ID_{LP_m}(K)$: Assume that $ID_3(K) < ID_{LP_m}(K)$. Then we can find a contradiction.

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- $ID_3(K) \leq ID_{LP_m}(K)$: Trivial since $I \models_{LP_m} K \Rightarrow I \models_3 (K)$.

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- $ID_3(K) \geq ID_{LP_m}(K)$: Assume that $ID_3(K) < ID_{LP_m}(K)$. Then we can find a contradiction.
- $ID_3(K) \leq ID_{LP_m}(K)$: Trivial since $I \models_{LP_m} K \Rightarrow I \models_3 (K)$.
- $ID_4(K) \leq ID_Q(K)$: Trivial since $I \models_Q K \Rightarrow I \models_4 (K)$.

Partial Max-SAT Problem

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- Partial Max-SAT Problem:
  - Optimized Version of SAT problem
  - $P = (H, S)$
  - H : hard clauses, all must be satisfied
  - S : soft clauses, should be satisfied as many as possible
  - $\hat{I} = \arg\max_I |\{\gamma \mid \gamma \in S, I \models \gamma, I \models H\}|$.

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- Max-SAT Competition
  - http://www.maxsat.udl.cat/09/
  - http://www.maxsat.udl.cat/10/

Computing Inconsistency Degrees

- only KB as a set of clauses (CNF) considered
- consider ID_4 and ID_Q (Since $ID_3(K) = ID_4(K) = ID_{LP_m}(K) \leq ID_Q(K)$)

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Road Map

1. Multi-valued semantics $\Rightarrow$ 2-valued semantics.
2. Represent ID_i by 2-valued semantics.
3. $ID_i \Rightarrow$ partial Max-SAT problem.

4-valued Logic to 2-valued Logic

$$
\begin{array}{rcl}
K & = & \{\gamma_1, \gamma_2, \ldots, \gamma_n\} \\
\gamma & = & l_1 \vee \ldots \vee l_k \\
l & = & p \\
l & = & \neg p
\end{array}
\quad \Rightarrow \quad
\begin{array}{rcl}
4(K) & = & \{4(\gamma_1), 4(\gamma_2), \ldots, 4(\gamma_n)\} \\
4(\gamma) & = & 4(l_1) \vee \ldots \vee 4(l_k) \\
4(p) & = & +p \\
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Example

$$K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$$

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Example

$K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$

Remark

- $4(K)$ is a knowledge base over variables $\{+p, -p \mid p \in Var(K)\}$
- A 4-valued interpretation I can also be seen as a 2-valued interpretation on $\{+p, -p \mid p \in Var(K)\}$.

4-valued Logic to 2-valued Logic

Theorem ([?])

$$I \models_4 K \;\Leftrightarrow\; I \models 4(K)$$

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- $K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$

4-valued Logic to 2-valued Logic

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Example

- $K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$
- $I_1 = \{+p, -p, -q, +r\}$

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Example

- $K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$
- $I_1 = \{+p, -p, -q, +r\}$
- As 4-interpretation over $\{p, q, r\}$:
  $p^{I_1} = B, q^{I_1} = f, r^{I_1} = t.$

4-valued Logic to 2-valued Logic

Theorem ([?])

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- $K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$
- $I_1 = \{+p, -p, -q, +r\}$
- As 4-interpretation over $\{p, q, r\}$:
  $p^{I_1} = B, q^{I_1} = f, r^{I_1} = t.$
- As 2-interpretation over $\{+p, -p, +q, -q, +r, -r\}$:
  $+p^{I_1} = t, -p^{I_1} = t, -q^{I_1} = t, +r^{I_1} = t, +q^{I_1} = f, -r^{I_1} = f.$

4-valued Logic to 2-valued Logic

Theorem ([?])

$$I \models_4 K \ \Leftrightarrow\ I \models 4(K)$$

Example

- $K = \{\neg p, p \vee q, \neg q, r\} \quad \Rightarrow \quad 4(K) = \{-p, +p \vee +q, -q, +r\}$
- $I_1 = \{+p, -p, -q, +r\}$
- As 4-interpretation over $\{p, q, r\}$:
  $p^{I_1} = B, q^{I_1} = f, r^{I_1} = t.$
- As 2-interpretation over $\{+p, -p, +q, -q, +r, -r\}$:
  $+p^{I_1} = t, -p^{I_1} = t, -q^{I_1} = t, +r^{I_1} = t, +q^{I_1} = f, -r^{I_1} = f.$
- We have $I_1 \models_4 K$ and $I_1 \models 4(K)$.

Representing ID_4 by 2-valued logic

$$ID_4(K, I) = \frac{|\{p \mid p^I = B, p \in Var(K)\}|}{|Var(K)|},$$

$$ID_4(K) = \min_{I \models_4 K} ID_4(K, I)$$

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$$\Downarrow$$

$$ID_4(K, I) = \frac{|\{p \mid +p^I = t \wedge -p^I = t, p \in Var(K)\}|}{|Var(K)|};$$

$$ID_4(K) = \min_{I \models 4(K)} ID_4(K, I) .$$

$ID_4 \Rightarrow$ Partial Max-SAT Problem

$$
\begin{aligned}
& min_{I \models 4(K)} |\{p \mid p \in Var(K), +p^I = t \wedge -p^I = t\}| \\
=& min_{I \models 4(K)} |\{p \mid p \in Var(K), (\neg + p \vee \neg - p)^I = f\}| \\
=& max_{I \models 4(K)} |\{p \mid p \in Var(K), (\neg + p \vee \neg - p)^I = t\}|.
\end{aligned}
$$

$I \models 4(K) \Rightarrow$ Hard constraints

$max|...| \Rightarrow$ Soft Constraints

$ID_4 \Rightarrow$ Partial Max-SAT Problem

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& min_{I \models 4(K)} |\{p \mid p \in Var(K), +p^I = t \wedge -p^I = t\}| \\
= & min_{I \models 4(K)} |\{p \mid p \in Var(K), (\neg + p \vee \neg - p)^I = f\}| \\
= & max_{I \models 4(K)} |\{p \mid p \in Var(K), (\neg + p \vee \neg - p)^I = t\}|.
\end{aligned}
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$$
\begin{aligned}
I \models 4(K) & \Rightarrow \text{Hard constraints} \\
max|...| & \Rightarrow \text{Soft Constraints}
\end{aligned}
$$

Definition

Given $K = \{\gamma_1, \ldots, \gamma_n\}$, the corresponding partial Max-SAT problem for ID_4, written $P_4(K) = (H_4(K), S_4(K))$, is defined as follows:

$$
\begin{aligned}
H_4(K) &= \{4(\gamma) \mid \gamma \in K\}; \\
S_4(K) &= \{\neg + p \vee \neg - p \mid p \in Var(K)\}.
\end{aligned}
$$

$ID_4 \Rightarrow$ Partial Max-SAT Problem

Theorem

Suppose I is a solution to the partial Max-SAT problem $P_4(K)$. Let $b = |\{p \mid +p^I = t \wedge -p^I = t\}|$ and $m = |Var(K)|$. Then $ID_4(K) = b/m$.

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Example

- $K = \{\neg p, p \vee q, \neg q, r\}$

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Example

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- $4(K) = \{-p, +p \vee +q, -q, +r\}$

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- $K = \{\neg p, p \vee q, \neg q, r\}$
- $4(K) = \{-p, +p \vee +q, -q, +r\}$
- $P_4(K) = (H_4(K), S_4(K))$

  $H_4(K) = \{-p, +p \vee +q, -q, +r\}$

  $S_4(K) = \{\neg + p \vee \neg - p, \neg + q \vee \neg - q, \neg + r \vee \neg - r\}$

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  $S_4(K) = \{\neg + p \vee \neg - p, \neg + q \vee \neg - q, \neg + r \vee \neg - r\}$
- One optimized solution I:

  $+p^I = t, -p^I = t, +q^I = f, -q^I = t, +r^I = t, -r^I = f.$

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- One optimized solution I:
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- Corresponding 4-model : $p^I = B, q^I = f, r^I = t.$

$ID_4 \Rightarrow$ Partial Max-SAT Problem

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Example

- $K = \{\neg p, p \vee q, \neg q, r\}$
- $4(K) = \{-p, +p \vee +q, -q, +r\}$
- $P_4(K) = (H_4(K), S_4(K))$

  $H_4(K) = \{-p, +p \vee +q, -q, +r\}$

  $S_4(K) = \{\neg + p \vee \neg - p, \neg + q \vee \neg - q, \neg + r \vee \neg - r\}$
- One optimized solution I:

  $+p^I = t, -p^I = t, +q^I = f, -q^I = t, +r^I = t, -r^I = f.$
- Corresponding 4-model : $p^I = B, q^I = f, r^I = t.$
- $ID_4(K) = 1/3$

Algorithm 1 Computing ID_4 by Partial MAX-SAT Solver

procedure $ID_4(K)$
    $P \leftarrow \{\}$
    $m \leftarrow |Var(K)|$
    for all Clause $\gamma \in K$ **do**
        $P.addHardClause(4(\gamma))$
    end for
    for all Variable $p \in Var(K)$ **do**
        $P.addSoftClause(\neg + p \vee \neg - p)$
    end for
    $I \leftarrow \text{PartialMaxSATSolver}(P)$
    $b = |\{p | +p^I = t \wedge -p^I = t\}|$
    return b/m
end procedure

Computing ID_Q

1. QC semantics $\Rightarrow$ 2-valued semantics.
2. Represent ID_Q by 2-valued semantics.
3. ID_Q $\Rightarrow$ partial Max-SAT problem.

Computing ID_Q

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$Q(l_1 \vee \ldots \vee l_n) = \bigvee_{i=1}^{n}(+l_i \wedge \neg - l_i) \vee \bigwedge_{i=1}^{n}(+l_i \wedge -l_i)$ **[?]**

2 Represent ID_Q by 2-valued semantics.

3 $ID_Q \Rightarrow$ partial Max-SAT problem.

Computing ID_Q

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2. Represent ID_Q by 2-valued semantics.
   $ID_Q(K) = \min_{I \models Q(K)} ID_Q(K, I)$.
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Computing ID_Q

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   Problem: $Q(l_1 \vee \ldots \vee l_n)$ can not keep CNF!
   Solution: Introduce new variables and convert it to CNF

Algorithm 2 Computing ID_Q by Partial Max-SAT Solver

procedure $ID_Q(K)$
  $P \leftarrow \{\}$
  $m \leftarrow |Var(K)|$
  for all Clause $\gamma = \{l_1, \ldots, l_n\} \in K$ **do**
    $P.addHardClause(y_1 \vee \ldots \vee y_n \vee z)$
    for $i = 1$ to n **do**
      $P.addHardClause(\neg y_i \vee +l_i)$
      $P.addHardClause(\neg y_i \vee \neg - l_i)$
      $P.addHardClause(\neg z \vee +l_i)$
      $P.addHardClause(\neg z \vee -l_i)$
    end for
  end for
  for all $p \in Var(K)$ **do**
    $P.addSoftClause(\neg +p \vee \neg -p)$
  end for
  $I \leftarrow$ PartialMaxSATSolver(P)
  $b = |\{p \mid +p^I = t \wedge -p^I = t\}|$
  return b/m
end procedure

Evaluation

- Data Set:
  - SAT benchmark SATLIB `http://www.satlib.org`
  - automotive product configuration

Evaluation

- Data Set:
  - SAT benchmark SATLIB `http://www.satlib.org`
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- Partial Max-SAT Solvers:
  - SAT4j MaxSAT
  - MsUncore
  - Clone

Instance				Encoding Algorithm		
name	#V	#C	ID_4	sat4j	MsUncore	clone
uuf50-0101	50	218	0.02000	0.396	0.026	1.119
uuf50-0102	50	218	0.02000	0.398	0.020	1.121
uuf50-0103	50	218	0.02000	0.450	0.044	1.142
uuf50-0104	50	218	0.02000	0.397	0.027	1.279
uuf75-011	75	325	0.01330	0.496	0.031	1.379
uuf75-012	75	325	0.01330	0.447	0.030	1.355
uuf75-013	75	325	0.01330	0.443	0.033	1.333
uuf75-014	75	325	0.01333	0.494	0.029	1.372
uuf100-0101	100	430	0.01000	0.545	0.045	1.748
uuf100-0102	100	430	0.01000	0.918	0.053	2.088
uuf100-0103	100	430	0.02000	3.951	2.592	*
C168_FW_SZ_107	1698	5401	0.00059	0.698	0.120	*
C168_FW_SZ_128	1698	5422	0.00059	0.601	0.090	13.191
C168_FW_SZ_41	1698	7489	0.00059	0.849	0.085	11.939

Table: Computing ID_4 by Encoding Algorithm

Instance				Encoding Algorithm		
name	#V	#C	ID_Q	sat4j	MSUnCore	clone
uuf50-0101	50	218	1.000	0.445	*	0.428
uuf50-0102	50	218	1.000	0.444	*	0.446
uuf50-0103	50	218	1.000	0.449	*	0.246
uuf50-0104	50	218	1.000	0.494	*	0.433
uuf75-011	75	325	1.000	0.544	*	0.434
uuf75-012	75	325	1.000	0.548	*	0.435
uuf75-013	75	325	1.000	0.455	*	1.338
uuf75-014	75	325	1.000	0.646	*	0.437
uuf100-0101	100	430	1.000	0.709	*	0.478
uuf100-0102	100	430	1.000	0.803	*	0.438
uuf100-0103	100	430	1.000	0.749	*	0.445
C168_FW_SZ_107	1698	5401	0.124	9.269	*	1.487
C168_FW_SZ_128	1698	5422	0.107	9.916	*	0.792
C168_FW_SZ_41	1698	7489	0.117	13.627	*	0.738

Table: Computing ID_Q by Encoding Algorithm

Conclusion & Future Work

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Conclusion:

- $ID_4(K) = ID_{LP_m}(K) = ID_3(K) \leq ID_Q(K)$
- $ID_4 \Rightarrow$ Partial Max-SAT
- $ID_Q \Rightarrow$ Partial Max-SAT

Conclusion & Future Work

Conclusion:

- $ID_4(K) = ID_{LP_m}(K) = ID_3(K) \leq ID_Q(K)$
- $ID_4 \Rightarrow$ Partial Max-SAT
- $ID_Q \Rightarrow$ Partial Max-SAT

Future Work:

- approximating inconsistency degrees
- Other encoding: Pseudo Problem for ID_Q
- Measure inconsistent Description Logic and Logic Program.

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Questions?